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Combinatorial Testing
with
Design of Experiments

An Executive Overview

11-CTDOEOVER-6A

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Introductions

- Name
- Organization
- Job Title/Responsibilities
- Experience in T&E, Combinatorial Testing, DOE, etc.

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Agenda

- **Some Basic Definitions and Terms**
- **Various Approaches to Testing Multiple Factors**
- **Design of Experiments (DOE): a Modern Approach to Combinatorial Testing**
- **Examples and Demonstration of a DOE**
- **Using DOE to Achieve Design Optimization**
- **Testing a Very Large Number of Factors**
- **Test Designs for Mixed Factors (Qualitative and Quantitative) and Mixed Levels**

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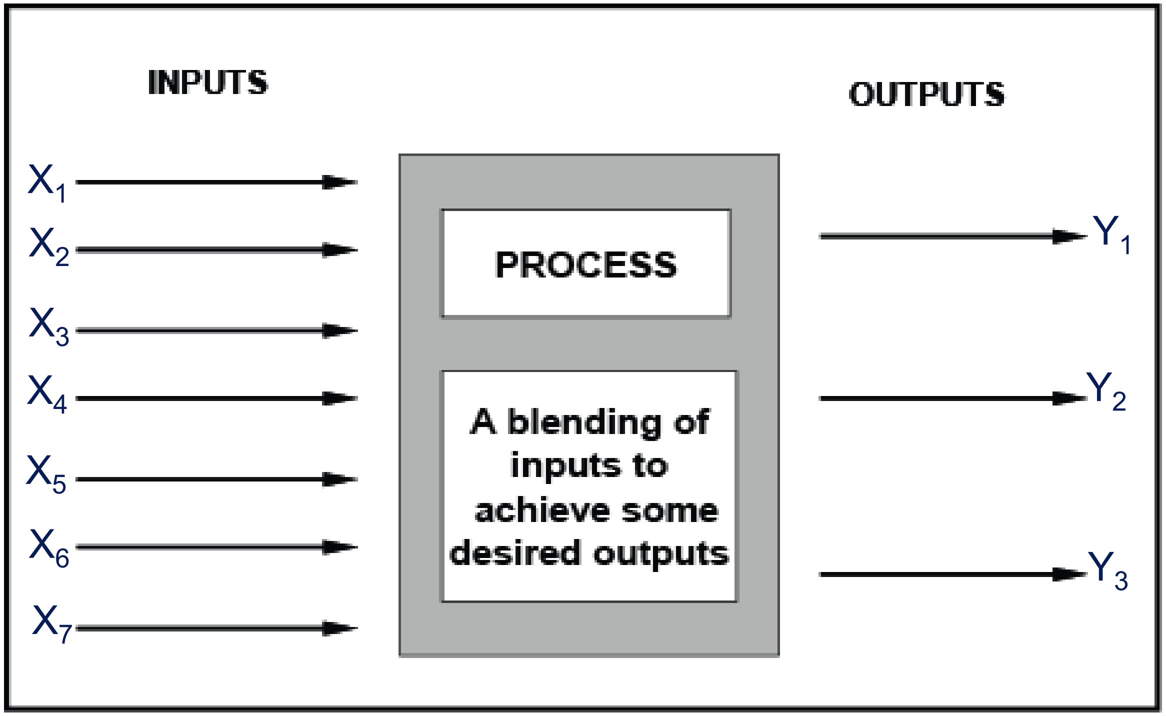
Definition of a Process


INPUTS

OUTPUTS

X_1

X_2

X_3

X_4

X_5

X_6

X_7

PROCESS

A blending of inputs to achieve some desired outputs

Y_1

Y_2

Y_3

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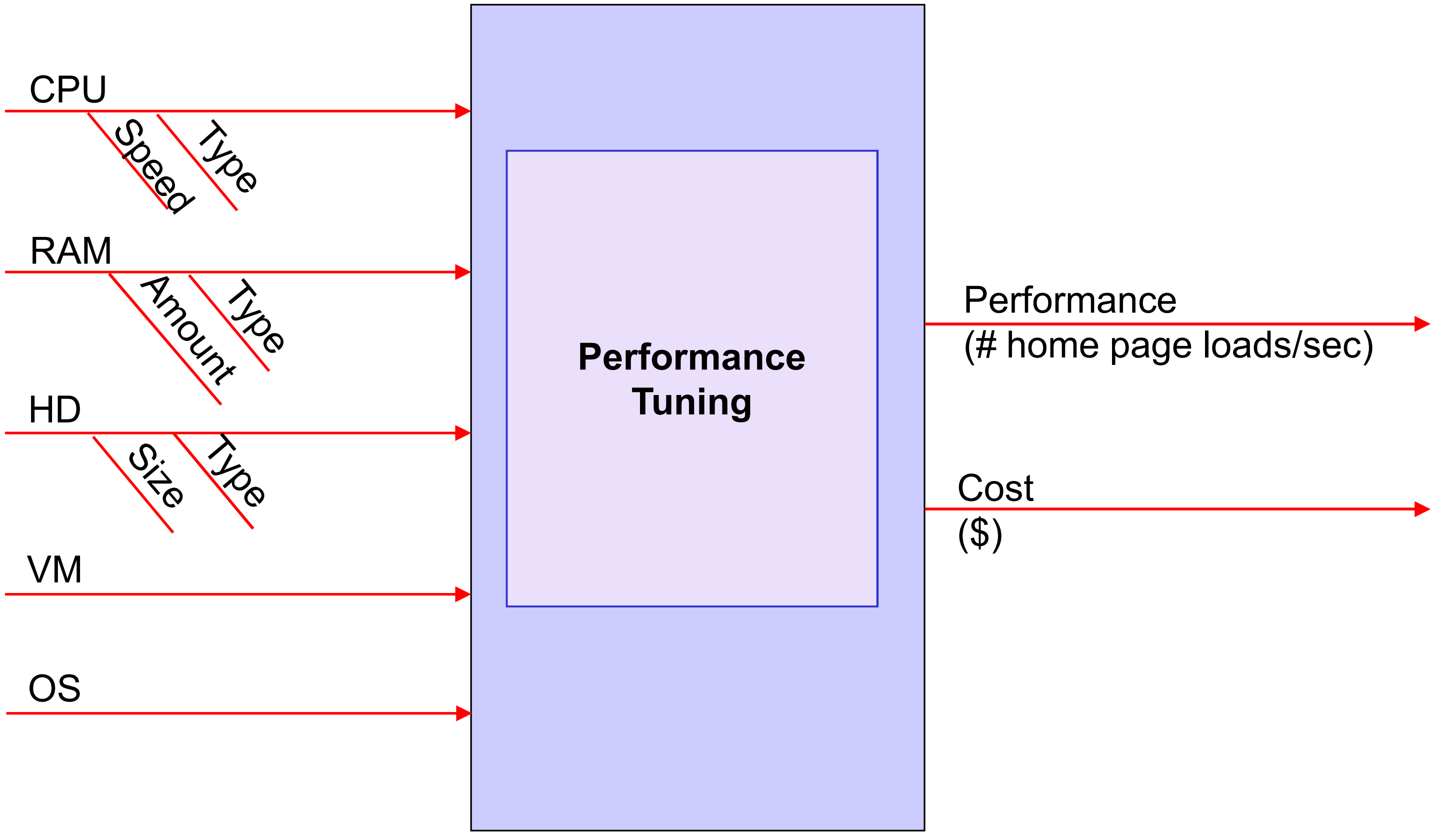
Web-Based Application Process



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Combinatorial Test Terminology

Y: Output, response variable, dependent variable

X: Input, factor, independent variable (a measurable entity that is purposely changed during an experiment)

Level: A unique value or choice of a factor (X)

Run: An experimental combination of the levels of the X's

Replication: Doing or repeating an experimental combination

Effect: The difference or impact on Y when changing X

Interaction: When the effect of one factor depends on the level of another factor

Performance Tuning Terminology

Factors/Inputs (X's)	Levels (Choices)	Performance/Outputs (Y's)
CPU Type	Itanium, Xeon	# home page loads/sec
CPU Speed	1 GHz, 2.5 GHz	Cost
RAM Amount	256 MB, 1.5 GB	
HD Size	50 GB, 500 GB	
VM	J2EE, .NET	
OS	Windows, Linux	

Which factors are important? Which are not?
Which combination of factor choices will maximize performance?
How do you know for sure? Show me the data.

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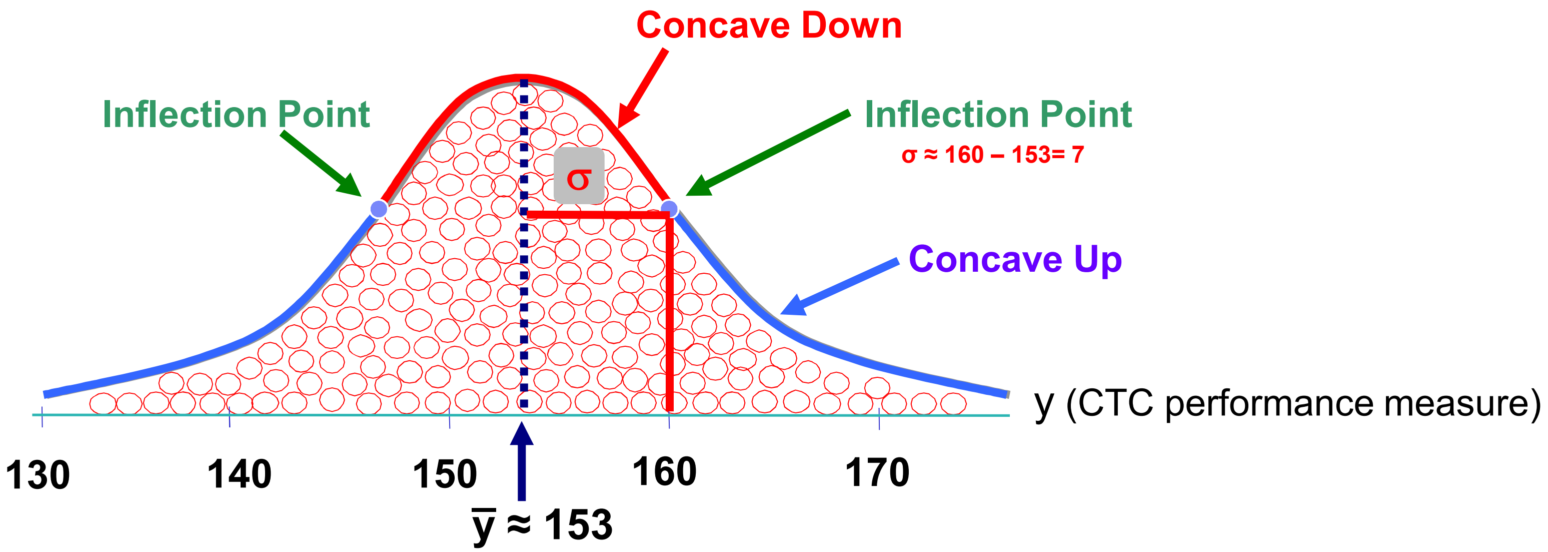
Graphical Meaning of $\bar{y}$ and σ

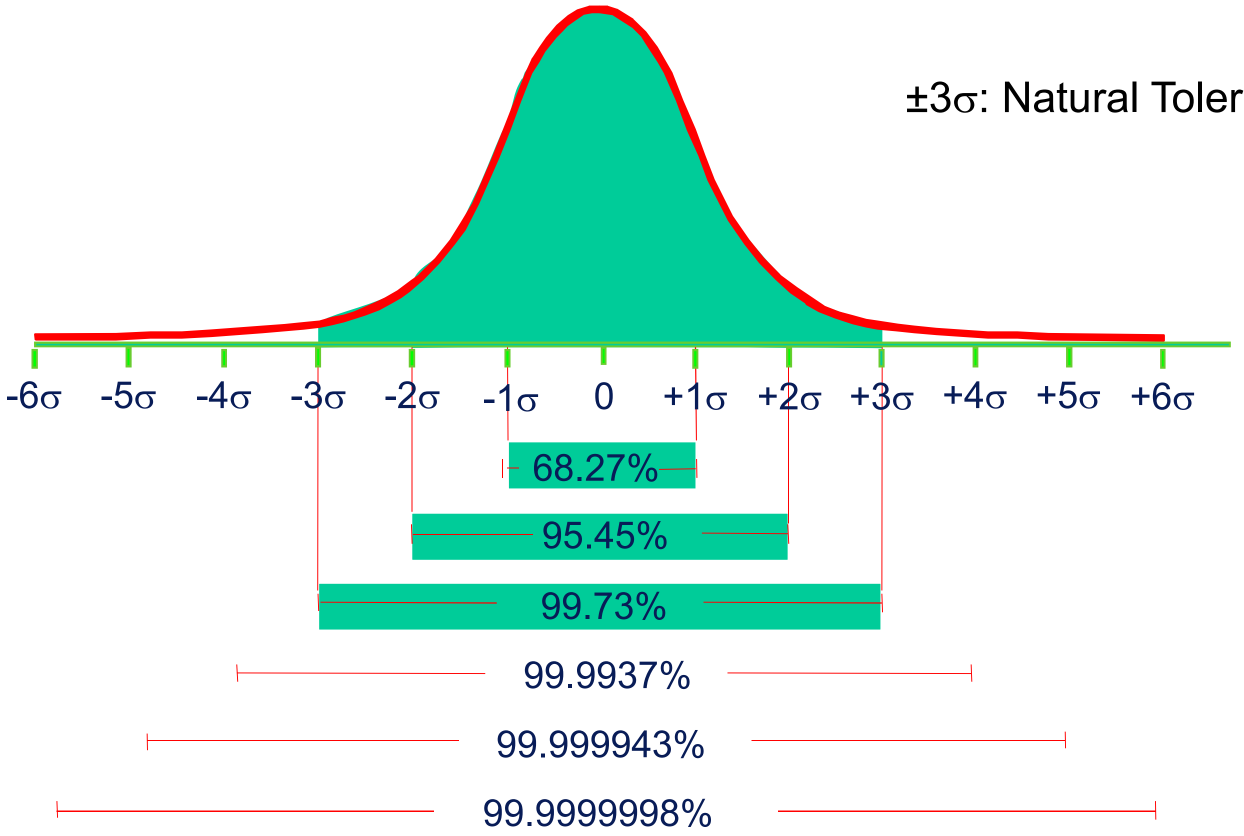
$\bar{y}$ = Average = Mean = Balance Point

σ = Standard Deviation



$\sigma \approx$ average distance of points from the centerline

Graphical View of Variation



Typical Areas under the Normal Curve

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Approaches to Testing Multiple Factors

- **Traditional Approaches**
  - One Factor at a Time (OFAT)
  - Oracle (Best Guess)
  - All possible combinations (full factorial)

- **Modern Approach**
  - Statistically designed experiments (DOE) … full factorial plus other selected DOE designs, depending on the situation

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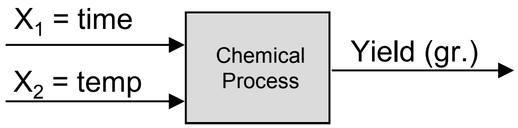
OFAT (One Factor at a Time)

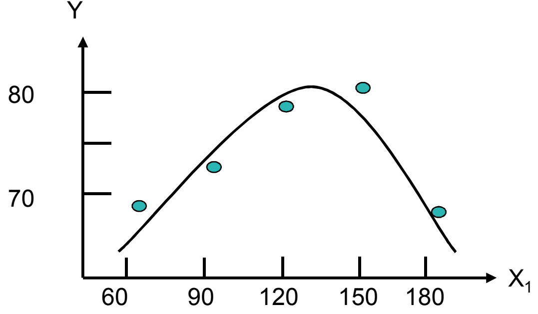

X_1 = time

X_2 = temp

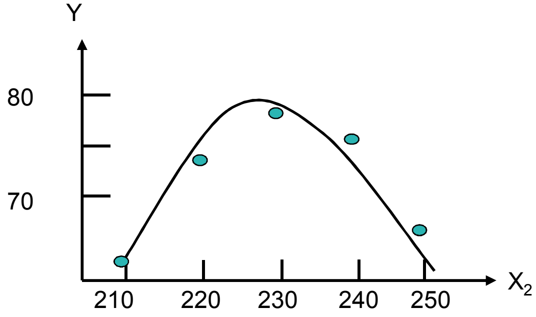
Chemical Process

Yield (gr.)

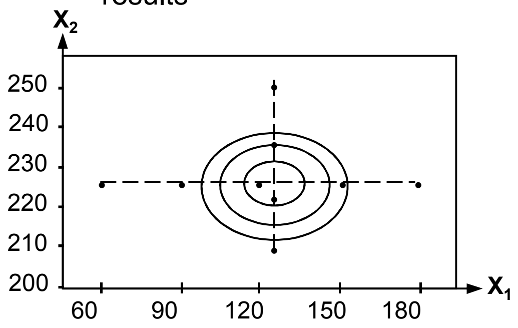
1. Hold X_2 constant and vary X_1
   Find the "best setting" for X_1



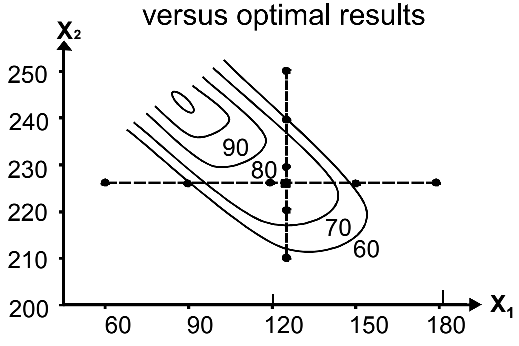
2. Hold X_1 constant at "best setting" and vary X_2.
   Find the "best setting" for X_2.



3. One factor at a time
   results



4. One factor at a time results
   versus optimal results



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The Good and Bad about OFAT

- **Good News**
  - Simple
  - Intuitive
  - The way we were originally taught

- **Bad News**
  - Will not be able estimate variable interaction effects
  - Will not be able to generate prediction models and thus not be able to optimize performance

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Oracle (Best Guess)

X1 = W = Wetting Agent (1=.07 ml; 2=none)

X2 = P = Plasticizer (1=1ml; 2=none)

X3 = E = Environment (1=Ambient Mixing; 2=Semi-Evacuated)

X4 = C = Cement (1=Portland Type III; 2=Calcium Aluminate)

X5 = A = Additive (1=No Reinforcement; 2=Steel)

Y = Strength of Lunar Concrete

Run	W	P	E	C	A	Y
1	1	2	1	1	1	5
2	1	1	1	1	1	6
3	2	2	1	1	1	5
4	2	1	1	1	2	6
5	1	2	2	2	2	7
6	1	1	2	2	2	8
7	2	2	2	2	2	10
8	2	1	2	2	1	11

Does factor C shift the average of Y?

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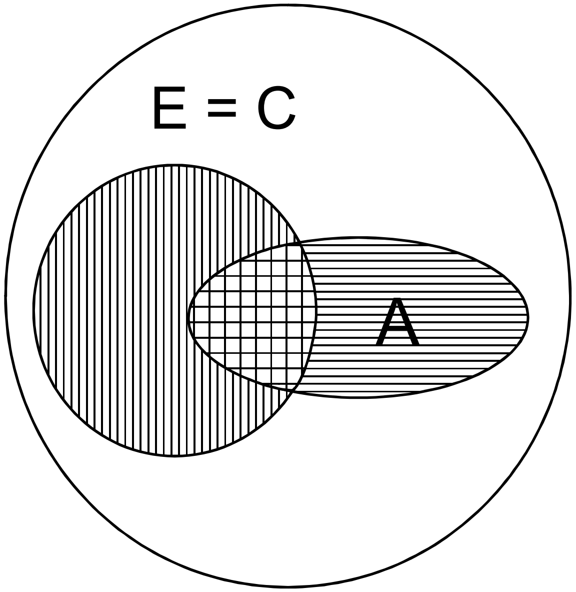
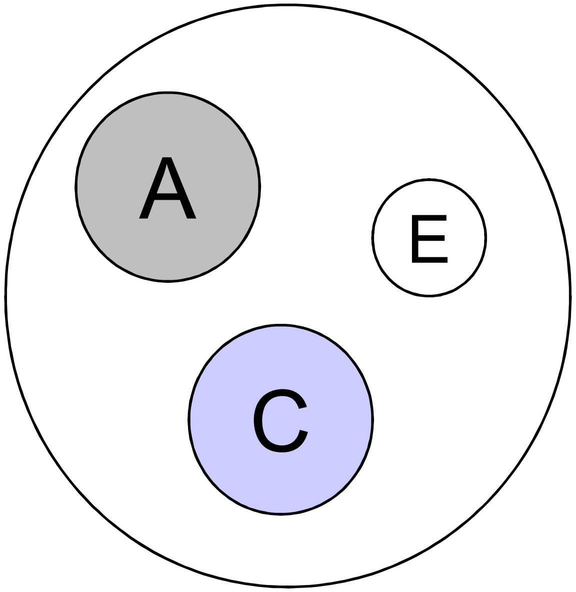
Evaluating the Effects of Variables on Y

What we have is:



What we need is a design to provide independent estimates of effects:



How do we obtain this independence of variables?

All Possible Combinations (Full Factorial)

MATRIX FORM

Example 1:

A (2 levels)
B (2 levels)

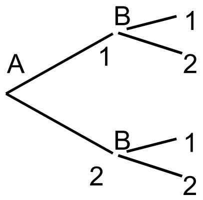
A	B
1	1
1	2
2	1
2	2

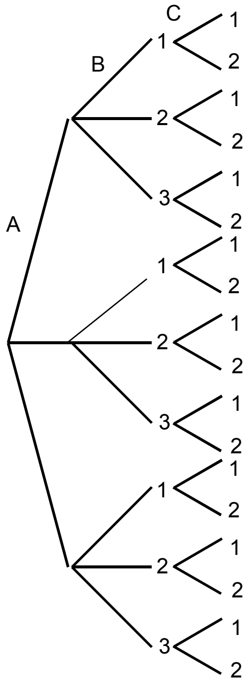
Example 2:

A (3 levels)
B (3 levels)
C (2 levels)

A	B	C
1	1	1
1	2	1
1	3	1
2	1	1
2	2	1
2	3	1
3	1	1
3	2	1
3	3	1
1	1	2
1	2	2
1	3	2
2	1	2
2	2	2
2	3	2
3	1	2
3	2	2
3	3	2

TREE DIAGRAM





Design of Experiments (DOE)

- An optimal data collection methodology
- “Interrogates” the process
- Used to identify important relationships between input and output factors
- Identifies important interactions between process variables
- Can be used to optimize a process
- Changes “I think” to “I know”

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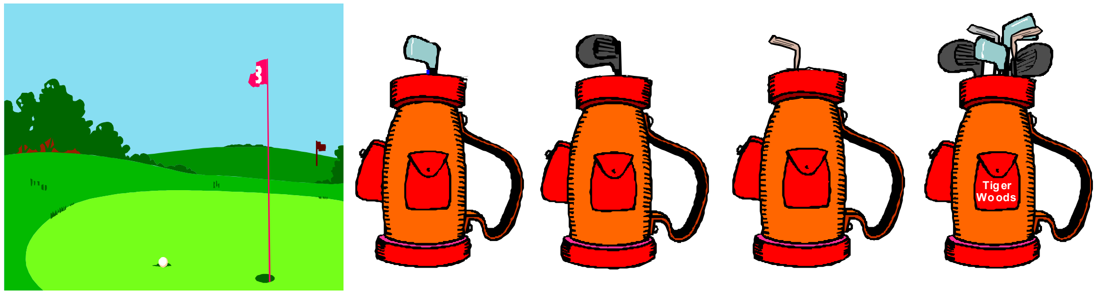
Important Contributions From:

	TAGUCHI	SHAININ	CLASSICAL	BLENDED APPROACH
Loss Function	*			*
Emphasis on Variance Reduction	*			*
Robust Designs	*			*
KISS	*	*		*
Simple Significance Tests		*		*
Component Swapping		*		
Multivariate Charts		*		*
Modeling			*	*
Sample Size			*	*
Efficient Designs			*	*
Optimization			*	*
Confirmation	*			*
Response Surface Methods			*	*



Which bag would a world class golfer prefer?

Statistically Designed Experiments (DOE):
Orthogonal or Nearly Orthogonal Designs

- FULL FACTORIALS (for small numbers of factors)
- FRACTIONAL FACTORIALS
- PLACKETT - BURMAN
- LATIN SQUARES
- HADAMARD MATRICES

} Taguchi Designs

- BOX - BEHNKEN DESIGNS
- CENTRAL COMPOSITE DESIGNS
- NEARLY ORTHOGONAL LATIN HYPERCUBE DESIGNS

SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

Run	Actual Settings			Coded Matrix			Responses
	(5, 10) A: Time	(70, 90) B: Temp	(100, 200) C: Press	(A) Time	(B) Temp	(C) Press	
1	5	70	100	-1	-1	-1	
2	5	70	200	-1	-1	+1	
3	5	90	100	-1	+1	-1	
4	5	90	200	-1	+1	+1	
5	10	70	100	+1	-1	-1	
6	10	70	200	+1	-1	+1	
7	10	90	100	+1	+1	-1	
8	10	90	200	+1	+1	+1	

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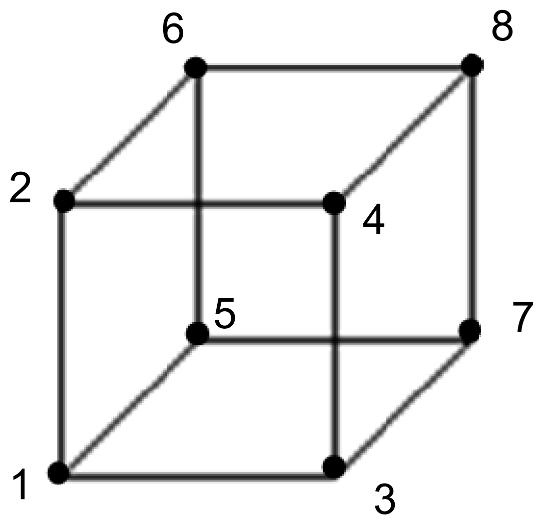
The Beauty of Orthogonality:
independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

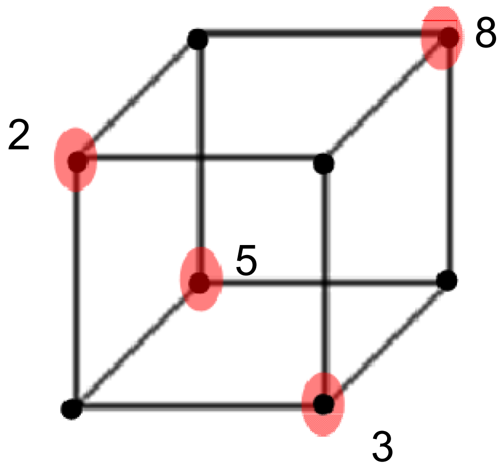
Run	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	-
2	-	-	+	+	-	-	+
3	-	+	-	-	+	-	+
4	-	+	+	-	-	+	-
5	+	-	-	-	-	+	+
6	+	-	+	-	+	-	-
7	+	+	-	+	-	-	-
8	+	+	+	+	+	+	+

Full Factorial vs. Fractional Factorial

(3 factors at 2 levels)



2^3 = 8-run Full Factorial Design



2^{3-1} = 4-run Fractional Factorial Design

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Screening Design

Taguchi L_{12} Design

Run	1	2	3	4	5	6	7	8	9	10	11
1	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	+	+	+	+	+	+
3	-	-	+	+	+	-	-	-	+	+	+
4	-	+	-	+	+	-	+	+	-	-	+
5	-	+	+	-	+	+	-	+	-	+	-
6	-	+	+	+	-	+	+	-	+	-	-
7	+	-	+	+	-	-	+	+	-	+	-
8	+	-	+	-	+	+	+	-	-	-	+
9	+	-	-	+	+	+	-	+	+	-	-
10	+	+	+	-	-	-	-	+	+	-	+
11	+	+	-	+	-	+	-	-	-	+	+
12	+	+	-	-	+	-	+	-	+	+	-

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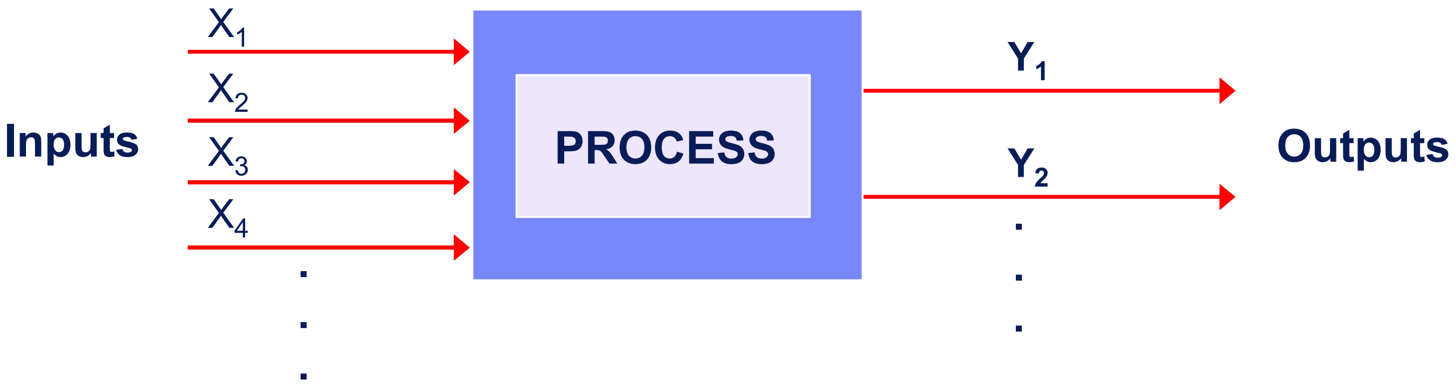
The Purpose of a Designed Experiment

Purposeful changes of the inputs (factors) in order to observe corresponding changes in the output (response).



Inputs

X_1

X_2

X_3

X_4

.
.
.

PROCESS

Y_1

Y_2

.
.
.

Outputs

Run	X_1	X_2	X_3	X_4	Y_1	Y_2	. . . . . .	$\bar{Y}$	S_Y
1									
2									
3									
.									
.									

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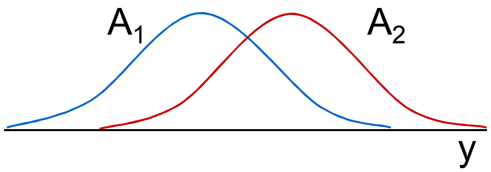
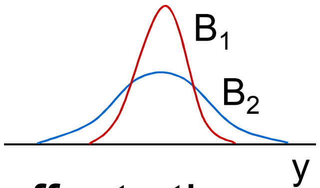
DOE Helps Determine How Inputs Affect Outputs

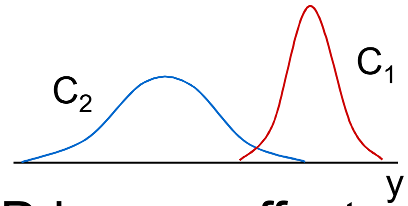
i) Factor A affects the average of y



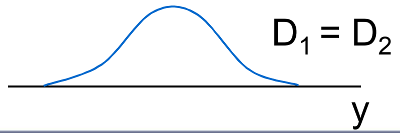
ii) Factor B affects the standard deviation of y



iii) Factor C affects the average and the standard deviation of y



iv) Factor D has no effect on y



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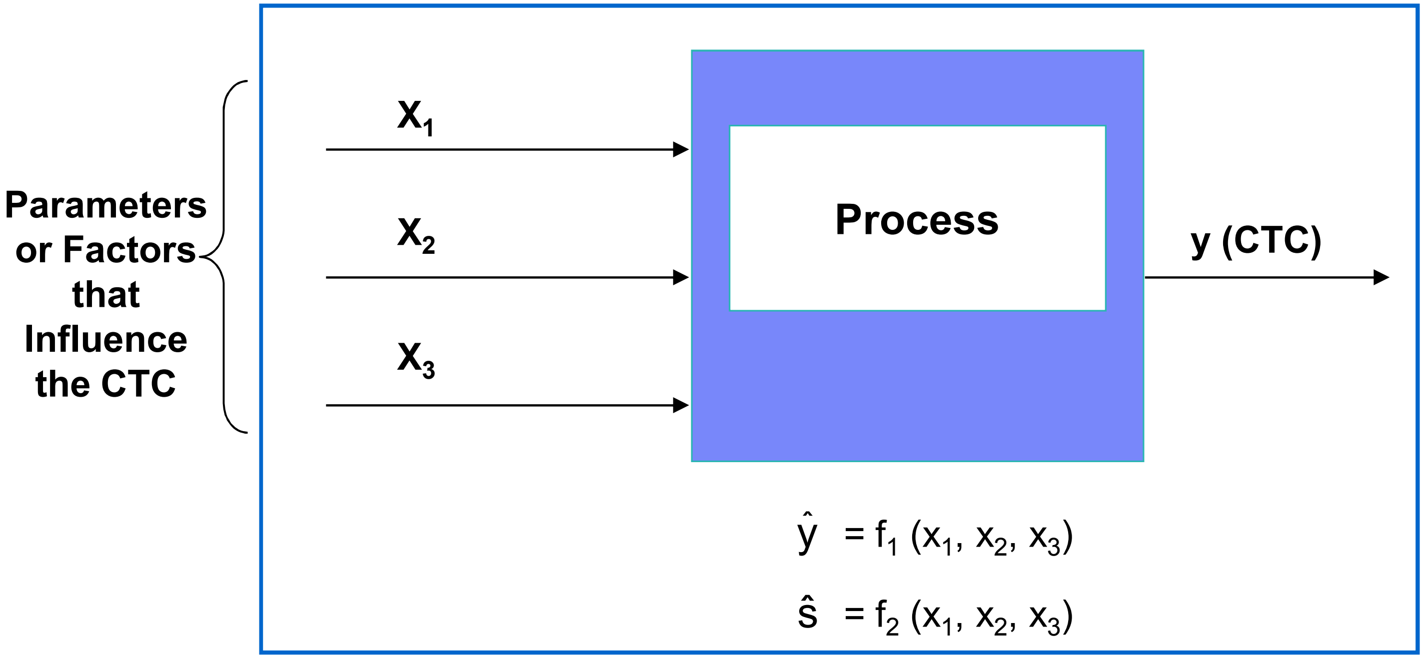
Transfer Functions



$\hat{y} = f_1(x_1, x_2, x_3)$

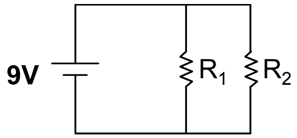
$\hat{s} = f_2(x_1, x_2, x_3)$

Where does the transfer function come from?

- **Exact transfer Function**
- **Approximations**
  - **DOE**
  - **Historical Data Analysis**
  - **Simulation**

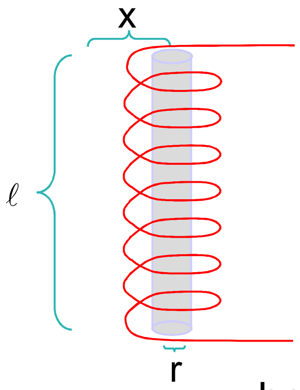
Exact Transfer Functions

- Engineering Relationships
  - V = IR
  - F = ma



The equation for current (I) through this DC circuit is defined by:

$$I=\frac{V}{\frac{R_1\cdot R_2}{R_1+R_2}}=\frac{V(R_1+R_2)}{R_1\cdot R_2}$$



The equation for magnetic force at a distance X from the center of a solenoid is:

$$H=\frac{NI}{2\ell}\left[\frac{.5\ell+x}{\sqrt{r^2+(.5\ell+x)^2}}+\frac{.5\ell-x}{\sqrt{r^2+(.5\ell-x)^2}}\right]$$

where

N: total number of turns of wire in the solenoid
I: current in the wire, in amperes
r: radius of helix (solenoid), in cm
ℓ: length of the helix (solenoid), in cm
x: distance from center of helix (solenoid), in cm
H: magnetizing force, in amperes per centimeter

Hierarchical Transfer Functions

$$Y = \text{Gross Margin} = \frac{\text{Gross Profit}}{\text{Gross Revenue}}$$

$$Y = f(y_1, y_2, y_3, y_4, y_5, y_6)$$

$$= \frac{(\overset{y_1}{\text{Rev}_{\text{equip}}} - \overset{y_2}{\text{COG}}) + (\overset{y_3}{\text{Rev}_{\text{post sales}}} - \overset{y_4}{\text{Cost}_{\text{post sales}}}) + (\overset{y_5}{\text{Rev}_{\text{fin}}} - \overset{y_6}{\text{Cost}_{\text{fin}}})}{y_1 + y_3 + y_5}$$

$$\overset{y_4}{\text{Cost}_{\text{post sales}}} = f(\overset{x_1}{\text{field cost}}, \overset{x_2}{\text{remote services}}, \overset{x_3}{\text{suppliers}})$$

$$x_1 = f(\text{direct labor, freight, parts, depreciation})$$

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Catapulting Power into Test and Evaluation

Statapult ® Catapult

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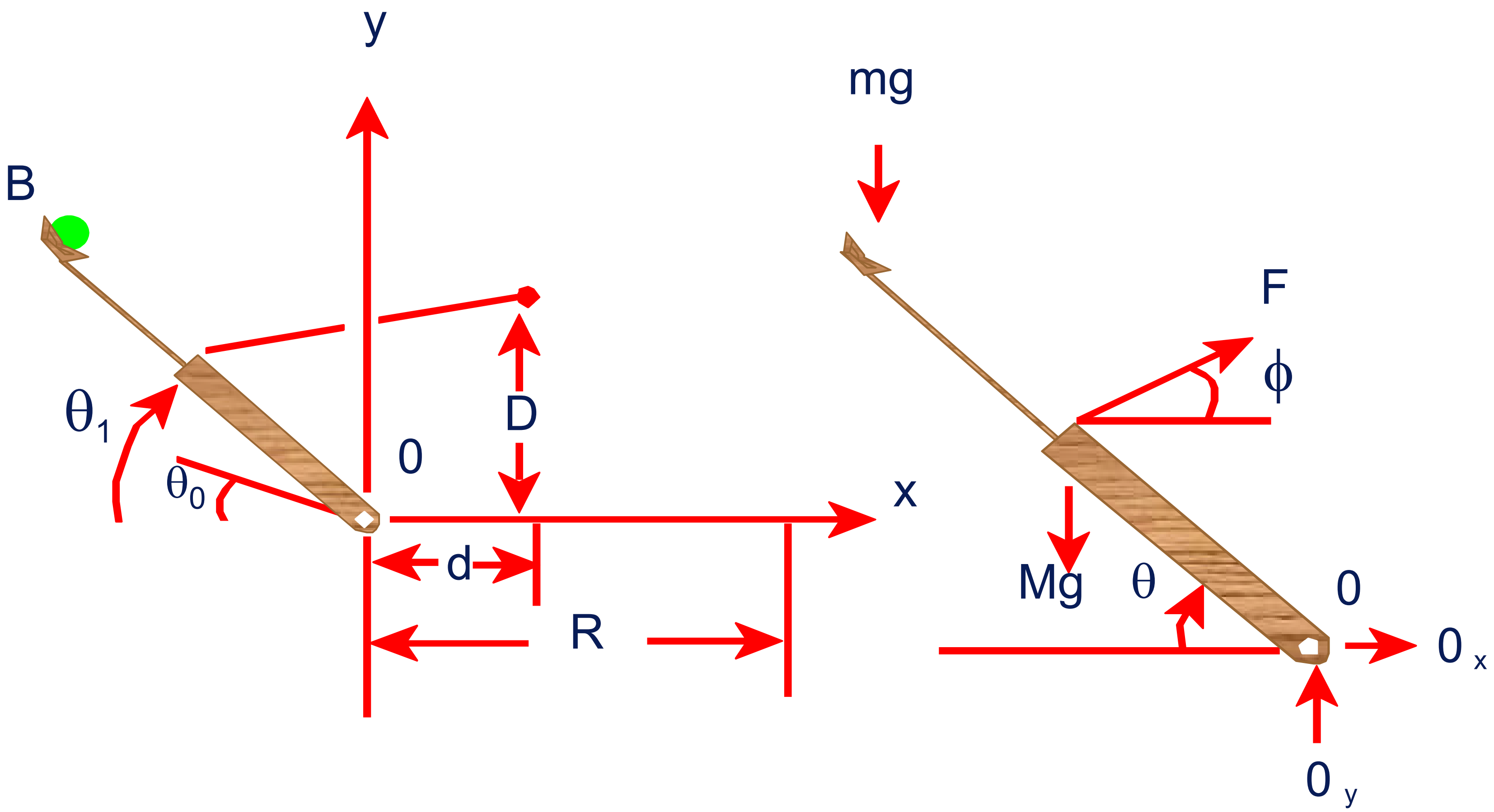
The Theoretical Approach



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The Theoretical Approach (cont.)

$$I_0\ddot{\theta} = r_F F(\theta)\sin\theta\cos\varphi - (Mgr_G + mgr_B)\sin\theta \qquad \tan\phi = \frac{D - r_F\sin\theta}{d + r_F\cos\theta},$$

$$\frac{1}{2}I_0\dot{\theta}^2 = r_F\int_{\theta_0}^{\theta} F(\theta)\sin\theta\cos\varphi\,d\theta - (Mgr_G + mgr_B)(\sin\theta - \sin\theta_0)$$

$$\frac{1}{2}I_0\dot{\theta}_1^2 = r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\varphi\,d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

$$x = v_B\cos\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}r_B\cos\theta_1 \qquad y = r_B\sin\theta_1 + v_B\sin\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}gt^2.$$

$$r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right) - \frac{g}{2V_B^2}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)} = 0.$$

$$\frac{gI_0}{4r_B}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)\left[r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right)\right]}$$

$$= r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\phi\,d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

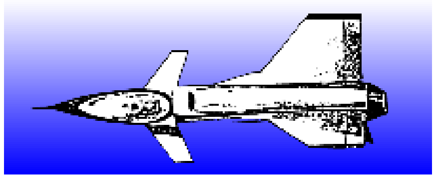
Statapult® DOE Demo

(The Empirical Approach)

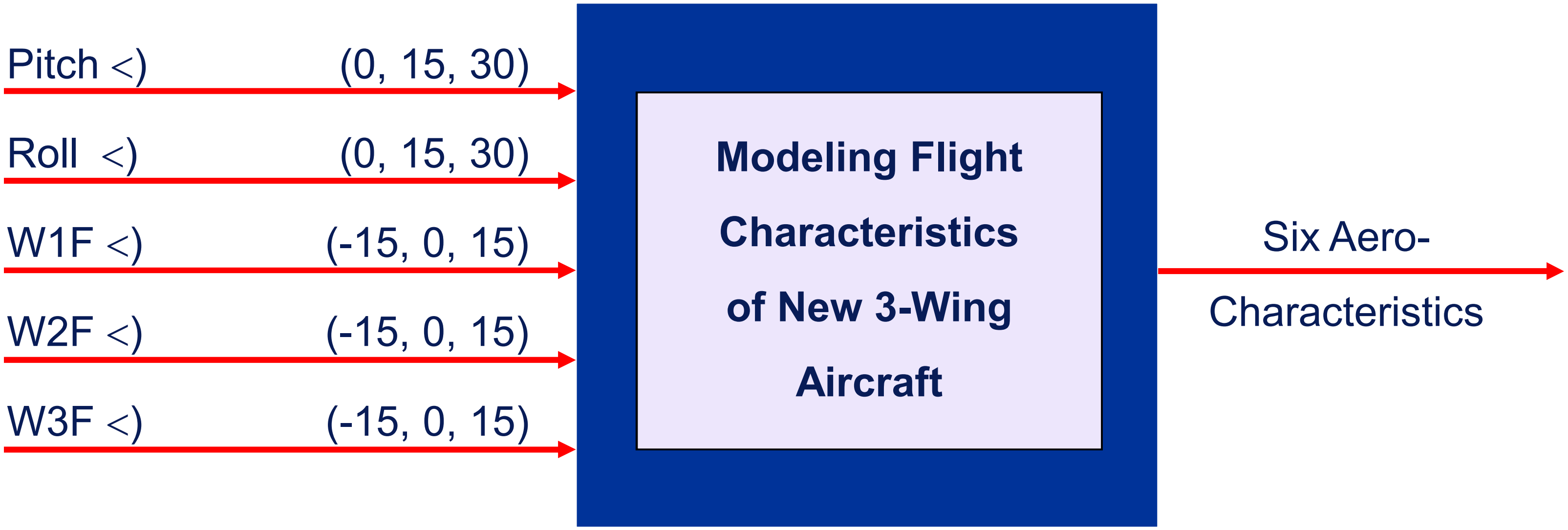
	Actual Factors		Coded Factors			Response Values			
Run	A	B	A	B	AB	Y_1	Y_2	$\bar{Y}$	S
1	144	2	-1	-1	+1				
2	144	3	-1	+1	-1				
3	160	2	+1	-1	-1				
4	160	3	+1	+1	+1				
		Avg –							
		Avg +							
		Δ							

Value Delivery: Reducing Time to Market for New Technologies

INPUT



OUTPUT



Pitch <) (0, 15, 30)

Roll <) (0, 15, 30)

W1F <) (-15, 0, 15)

W2F <) (-15, 0, 15)

W3F <) (-15, 0, 15)

Modeling Flight Characteristics of New 3-Wing Aircraft

Six Aero- Characteristics

- **Total # of Combinations = $3^5 = 243$**
- **Central Composite Design: n = 30**

Patent Holder: Dr. Bert Silich

Aircraft Equations

$$C_L = .233 + .008(P)^2 + .255(P) + .012(R) - .043(WD1) - .117(WD2) + .185(WD3) + .010(P)(WD3) - .042(R)(WD1) + .035(R)(WD2) + .016(R)(WD3) + .010(P)(R) - .003(WD1)(WD2) - .006(WD1)(WD3)$$

$$C_D = .058 + .016(P)^2 + .028(P) - .004(WD1) - .013(WD2) + .013(WD3) + .002(P)(R) - .004(P)(WD1) - .009(P)(WD2) + .016(P)(WD3) - .004(R)(WD1) + .003(R)(WD2) + .020(WD1)^2 + .017(WD2)^2 + .021(WD3)^2$$

$$C_Y = -.006(P) - .006(R) + .169(WD1) - .121(WD2) - .063(WD3) - .004(P)(R) + .008(P)(WD1) - .006(P)(WD2) - .008(P)(WD3) - .012(R)(WD1) - .029(R)(WD2) + .048(R)(WD3) - .008(WD1)^2$$

$$C_M = .023 - .008(P)^2 + .004(P) - .007(R) + .024(WD1) + .066(WD2) - .099(WD3) - .006(P)(R) + .002(P)(WD2) - .005(P)(WD3) + .023(R)(WD1) - .019(R)(WD2) - .007(R)(WD3) + .007(WD1)^2 - .008(WD2)^2 + .002(WD1)(WD2) + .002(WD1)(WD3)$$

$$C_{YM} = .001(P) + .001(R) - .050(WD1) + .029(WD2) + .012(WD3) + .001(P)(R) - .005(P)(WD1) - .004(P)(WD2) - .004(P)(WD3) + .003(R)(WD1) + .008(R)(WD2) - .013(R)(WD3) + .004(WD1)^2 + .003(WD2)^2 - .005(WD3)^2$$

$$C_e = .003(P) + .035(WD1) + .048(WD2) + .051(WD3) - .003(R)(WD3) + .003(P)(R) - .005(P)(WD1) + .005(P)(WD2) + .006(P)(WD3) + .002(R)(WD1)$$

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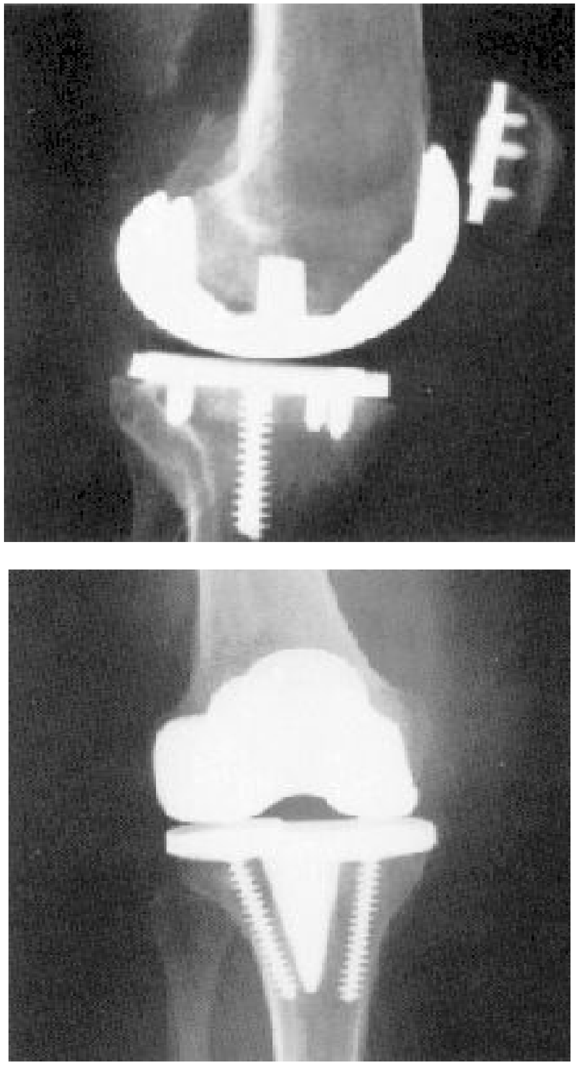
Fusing Titanium and Cobalt-Chrome



Courtesy Rai Chowdhary

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DOE “Market Research” Example

Suppose that, in the auto industry, we would like to investigate the following automobile attributes (i.e., factors), along with accompanying levels of those attributes:

A: Brand of Auto:	**-1 = foreign**		**+1 = domestic**
B: Auto Color:	**-1 = light**	**0 = bright**	**+1 = dark**
C: Body Style:	**-1 = 2-door**	**0 = 4-door**	**+1 = sliding door/hatchback**
D: Drive Mechanism:	**-1 = rear wheel**	**0 = front wheel**	**+1 = 4-wheel**
E: Engine Size:	**-1 = 4-cylinder**	**0 = 6-cylinder**	**+1 = 8-cylinder**
F: Interior Size:	**-1 ≤ 2 people**	**0 = 3-5 people**	**+1 ≥ 6 people**
G: Gas Mileage:	**-1 ≤ 20 mpg**	**0 = 20-30 mpg**	**+1 ≥ 30 mpg**
H: Price:	**-1 ≤ $20K**	**0 = $20-$40K**	**+1 ≥ $40K**

In addition, suppose the respondents chosen to provide their preferences to product profiles are taken based on the following demographic:

J: Age:	**-1 ≤ 25 years old**	**+1 ≥ 35 years old**
K: Income:	**-1 ≤ $30K**	**+1 ≥ $40K**
L: Education:	**-1 < BS**	**+1 ≥ BS**

DOE "Market Research" Example (cont.)

Question: **Choose the best design for evaluating this scenario**

Answer: **L_{18} design with attributes A - H in the inner array and factors J, K, and L in the outer array, resembling an L_{18} robust design, as shown below:**

									L	-	+	-	+	-	+	-	+		
									K	-	-	+	+	-	-	+	+		
									J	-	-	-	-	+	+	+	+		
Run*	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	$\bar{y}$	**s**
1	-	-	-	-	-	-	-	-											
2	-	-	0	0	0	0	0	0		Segmentation of the population or <u>Respondent Profiles</u>									
3	-	-	+	+	+	+	+	+											
4	-	0	-	-	0	0	+	+											
5	-	0	0	0	+	+	-	-											
6	-	0	+	+	-	-	0	0											
7	-	+	-	0	-	+	0	+											
8	-	+	0	+	0	-	+	-											
9	-	+	+	-	+	0	-	0											
10	+	-	-	+	+	0	0	-											
11	+	-	0	-	-	+	+	0											
12	+	-	+	0	0	-	-	+											
13	+	0	-	0	+	-	+	0											
14	+	0	0	+	-	0	-	+											
15	+	0	+	-	0	+	0	-											
16	+	+	-	+	0	+	-	0											
17	+	+	0	-	+	-	0	+											
18	+	+	+	0	-	0	+	-											

* 18 different product profiles

Modeling The Drivers of Turnover

① External Market Factors (Local Labor Market Conditions)

- Local Unemployment Rate
- Local Employment Alternatives
- Company's Market Share

② Organizational Characteristics and Practices

- Supervisor Stability
- Lateral / Upward Mobility
- Layoff Climate

③ Employee Attributes

- Time Since Last Promotion
- Education Level
- Job Stability History

Process of Deciding to Stay / Leave

Turnover Rate

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Google on DOE

(quotes* from Daryl Pregibon, Google Engineer)

"From a user's perspective, a query was submitted and results appear. From Google's perspective, the user has provided an opportunity to test something. What can we test? Well, there is so much to test that we have an Experiment Council that vets experiment proposals and quickly approves those that pass muster."

" We <u>evangelize</u> experimentation to the extent that we provide a mechanism for advertisers to run their own experiments.

. . . allows an advertiser to run a (full) factorial experiment on its web page. Advertisers can explore layout and content alternatives while Google randomly directs queries to the resulting treatment combinations. Simple analysis of click and conversion rates allows advertisers to explore a range of alternatives and their effect on user awareness and interest."

*** Taken From: *Statistics @ Google* in Amstat News, May 2011**

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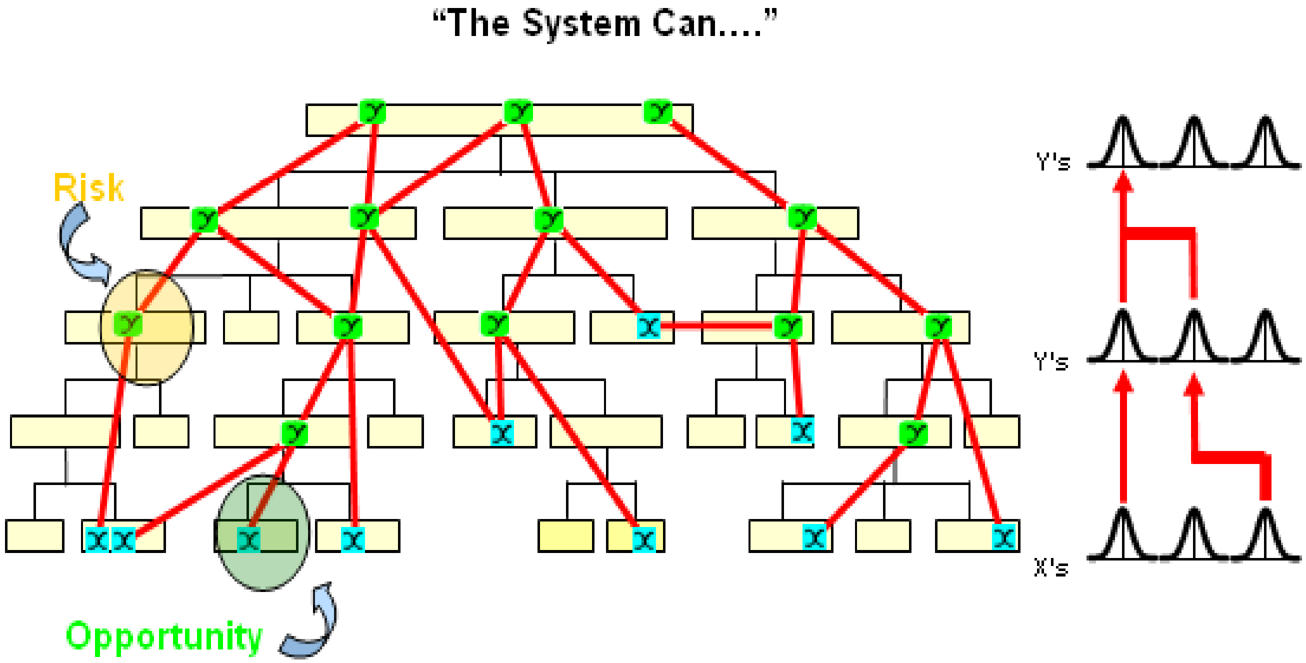
DOE Enables Critical Parameter Management

(CPM)

CPM is a systems engineering best practice that is extremely useful in managing, analyzing, and reporting technical product performance.



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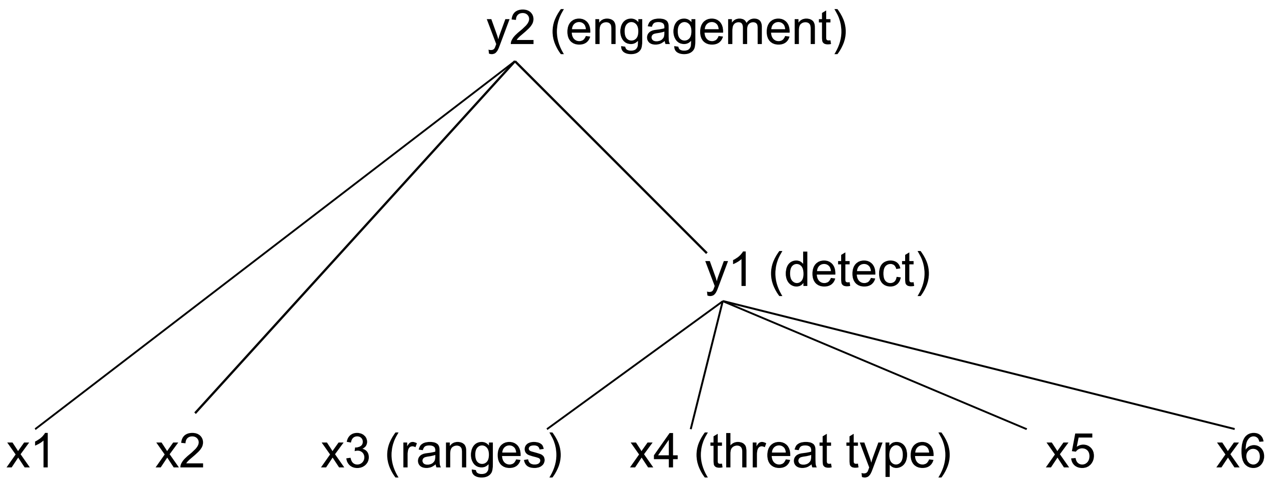
Critical Parameter Management and COIs

- A Critical Operational Issue (COI) is linked to operational effectiveness and suitability.
- It is typically phrased as a question, e.g.,

Will the system ***detect*** the ***threat*** in a ***combat environment*** at adequate ***range*** to allow for successful ***engagement***?



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DOE: the bridge to Design Optimization and Systematic Innovation

- **Expected Value Analysis**
- **Parameter (Robust) Design**

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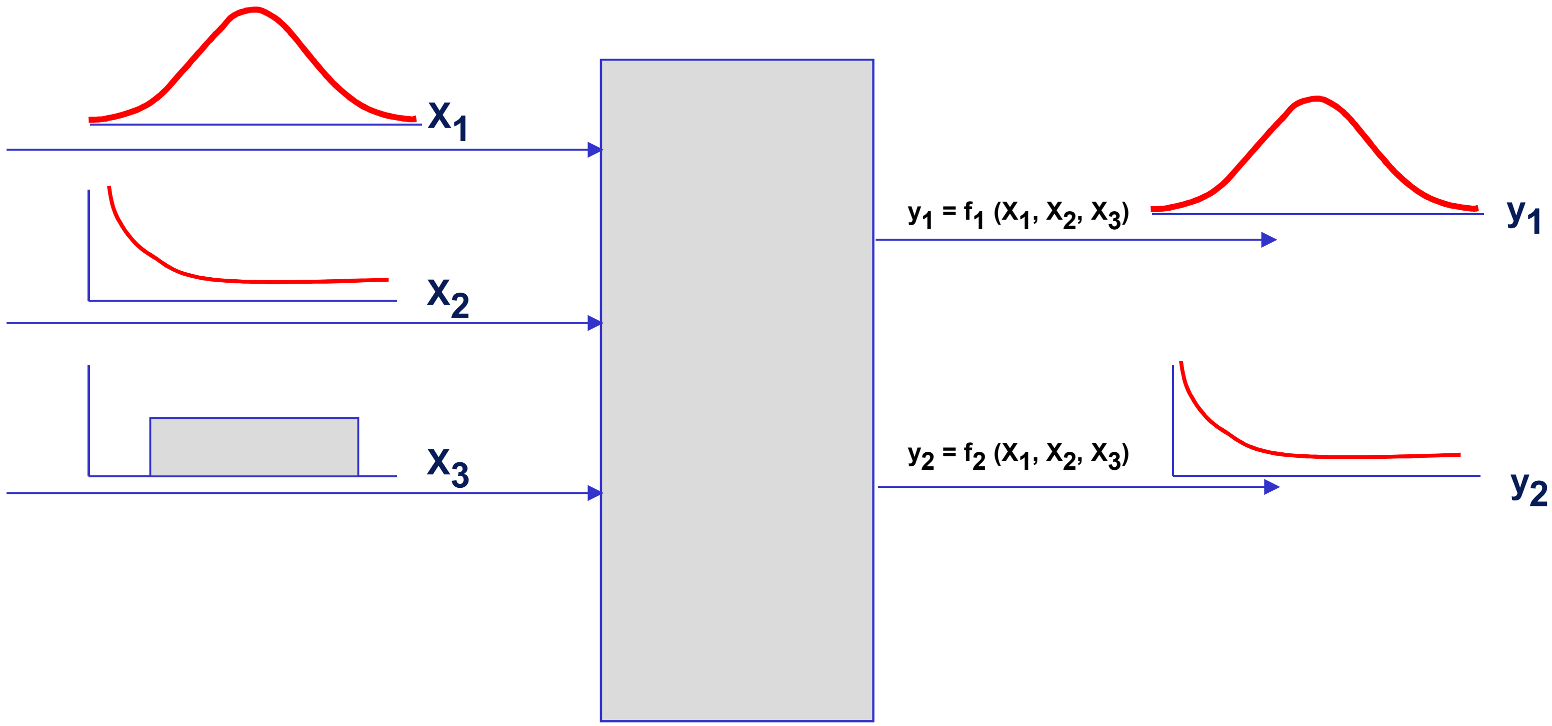
Expected Value Analysis (EVA)

EVA is the technique used to determine the characteristics of the output distribution (mean, standard deviation, and shape) when we have knowledge of (1) the input variable distributions and (2) the transfer functions.



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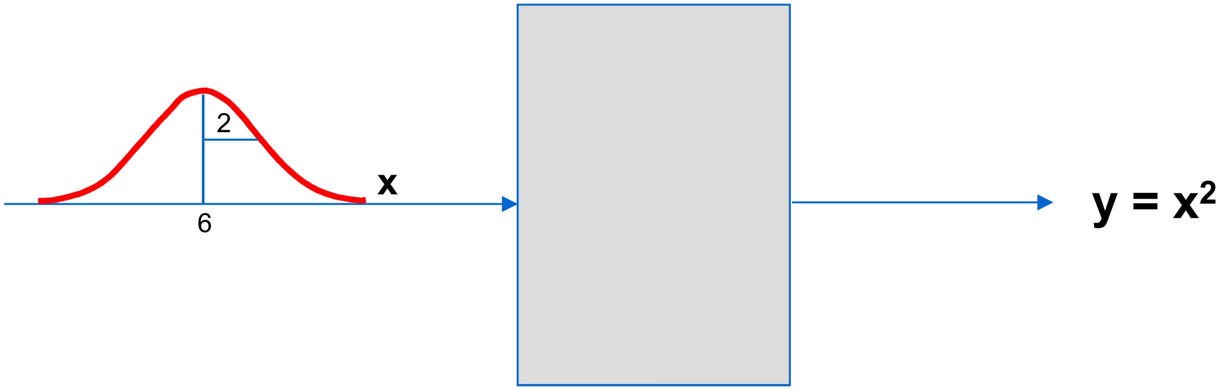
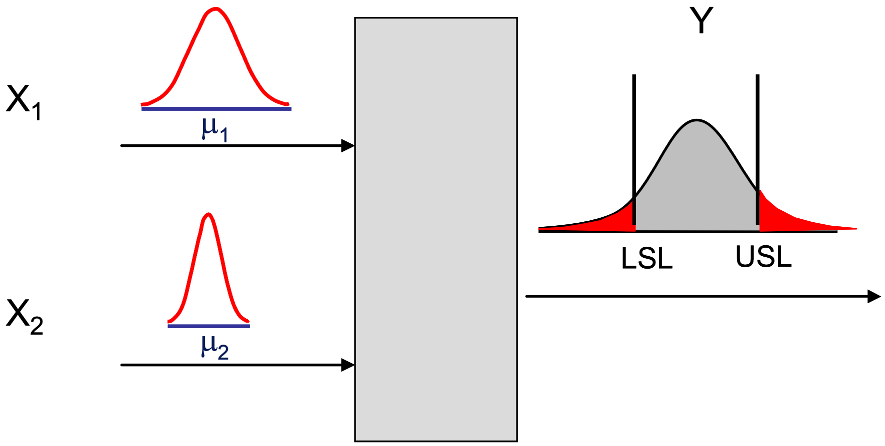
Expected Value Analysis Example



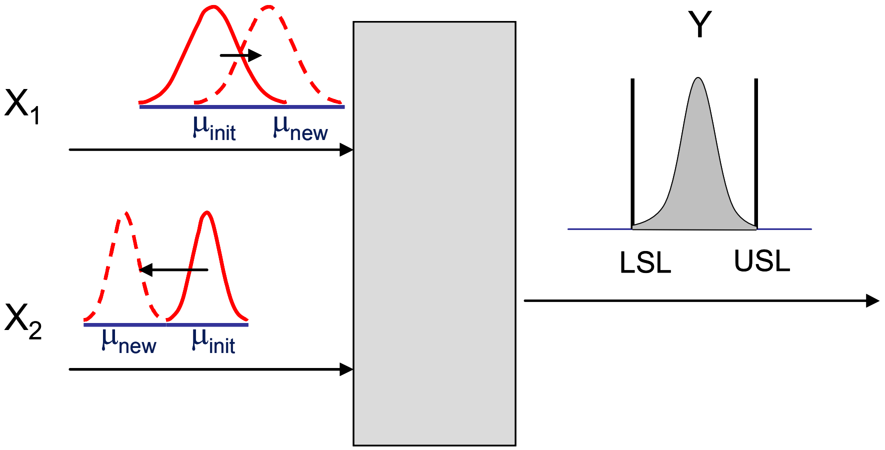
What is the mean or expected value of the y distribution?

What is the shape of the y distribution?

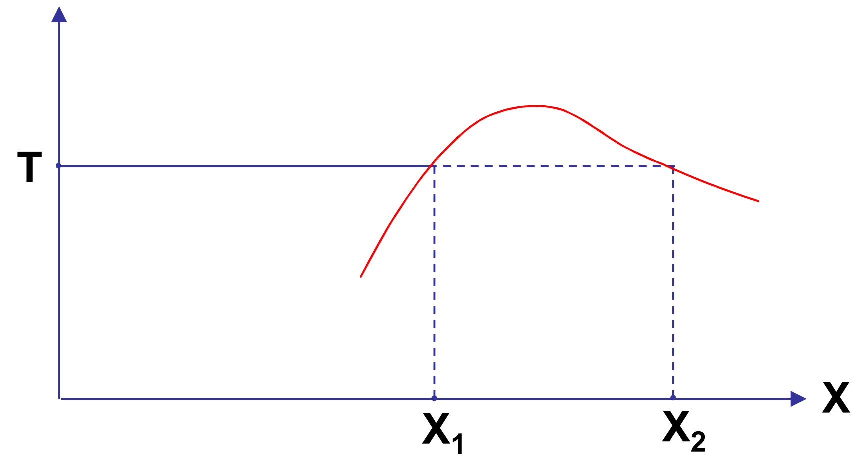
Parameter Design (Robust Design)



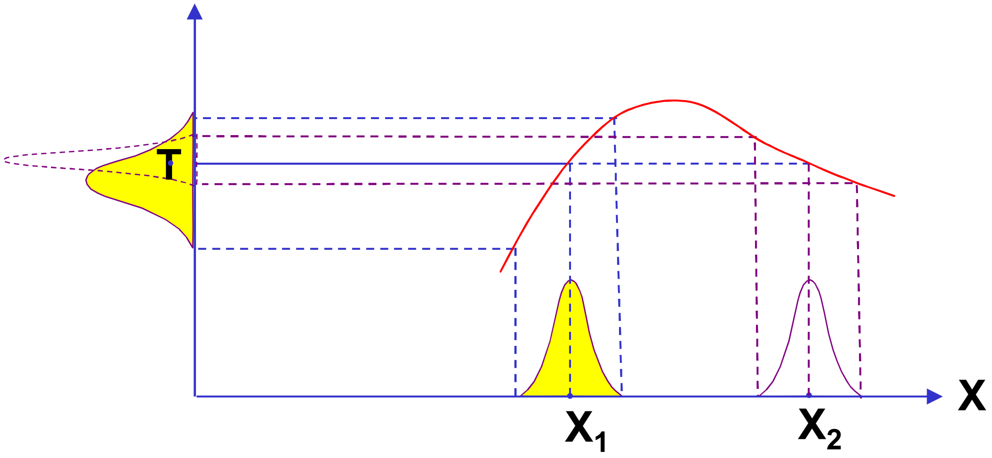
Process of finding the optimal mean settings of the input variables to minimize the resulting dpm.



Parameter Design (Robust Design)



If you're the designer, which setting for X do you prefer?



Changing the mean of an input may possibly reduce the output variation!

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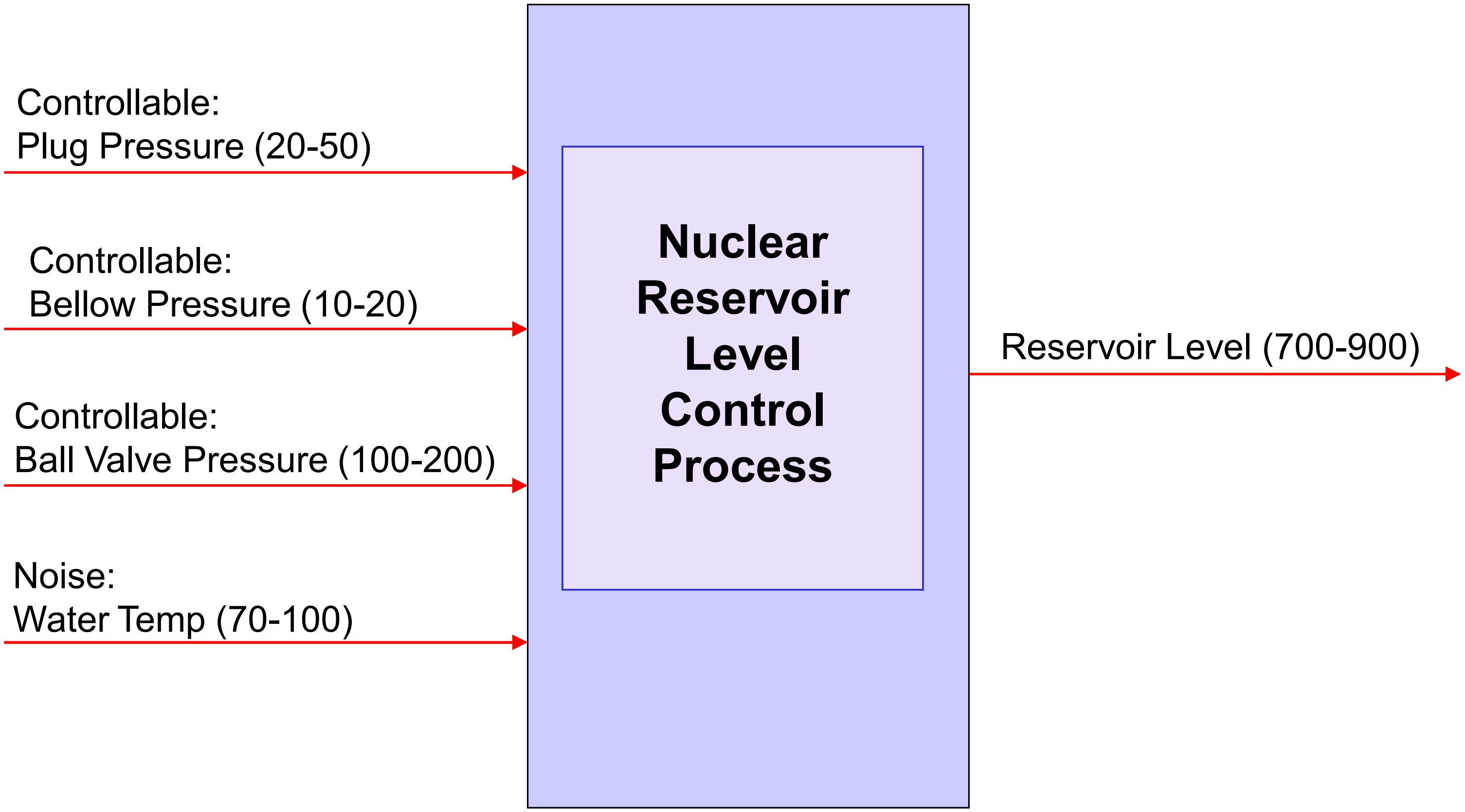
Robust (Parameter) Design Simulation* Example



Controllable:
Plug Pressure (20-50)

Controllable:
Bellow Pressure (10-20)

Controllable:
Ball Valve Pressure (100-200)

Noise:
Water Temp (70-100)

Nuclear Reservoir Level Control Process

Reservoir Level (700-900)

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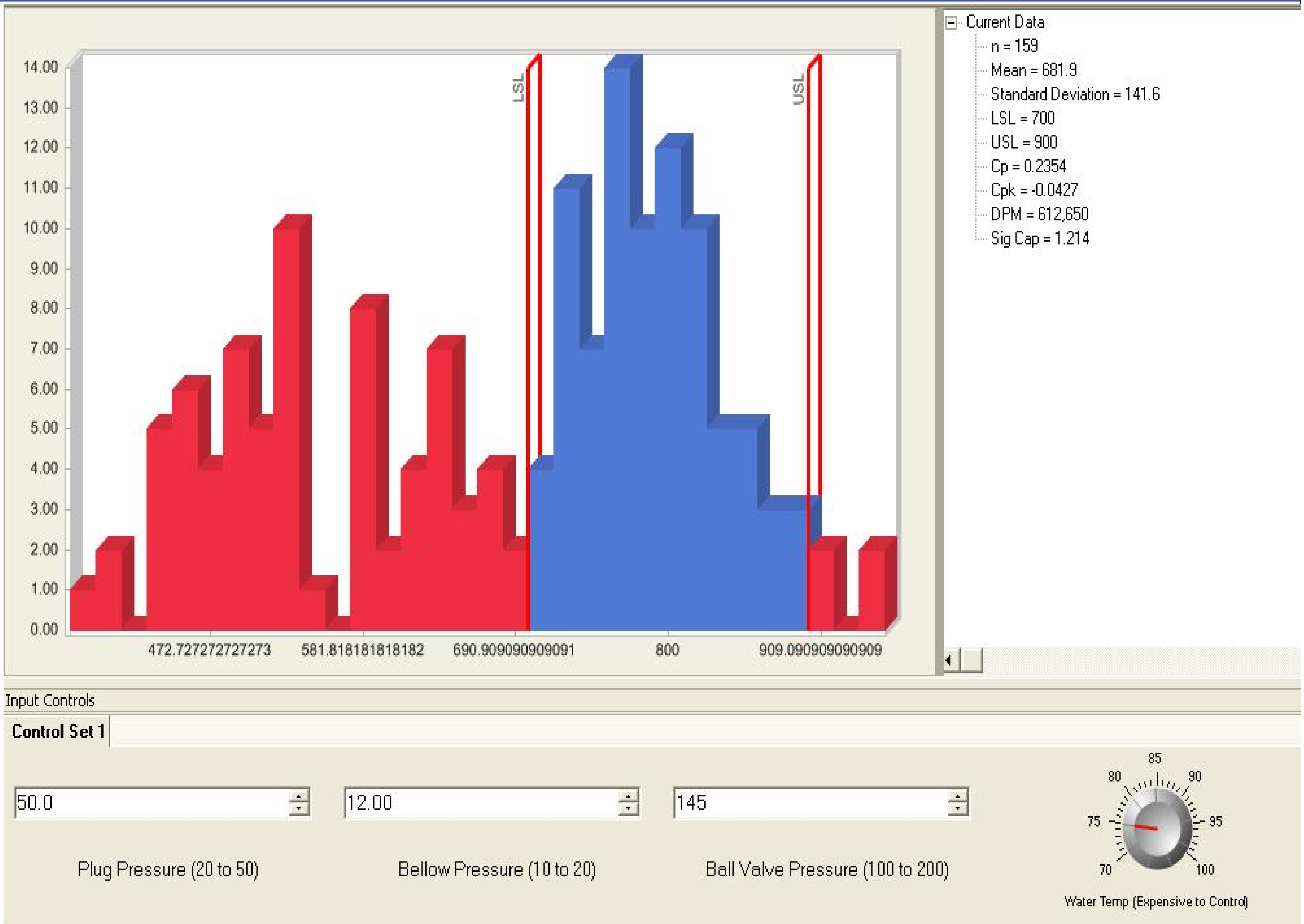
* From **SimWare Pro** by Digital Computations

Prior to Robust Design



Current Data
- n = 159
- Mean = 681.9
- Standard Deviation = 141.6
- LSL = 700
- USL = 900
- Cp = 0.2354
- Cpk = -0.0427
- DPM = 612,650
- Sig Cap = 1.214

Input Controls

Control Set 1

Plug Pressure (20 to 50): 50.0

Bellow Pressure (10 to 20): 12.00

Ball Valve Pressure (100 to 200): 145

Water Temp (Expensive to Control)

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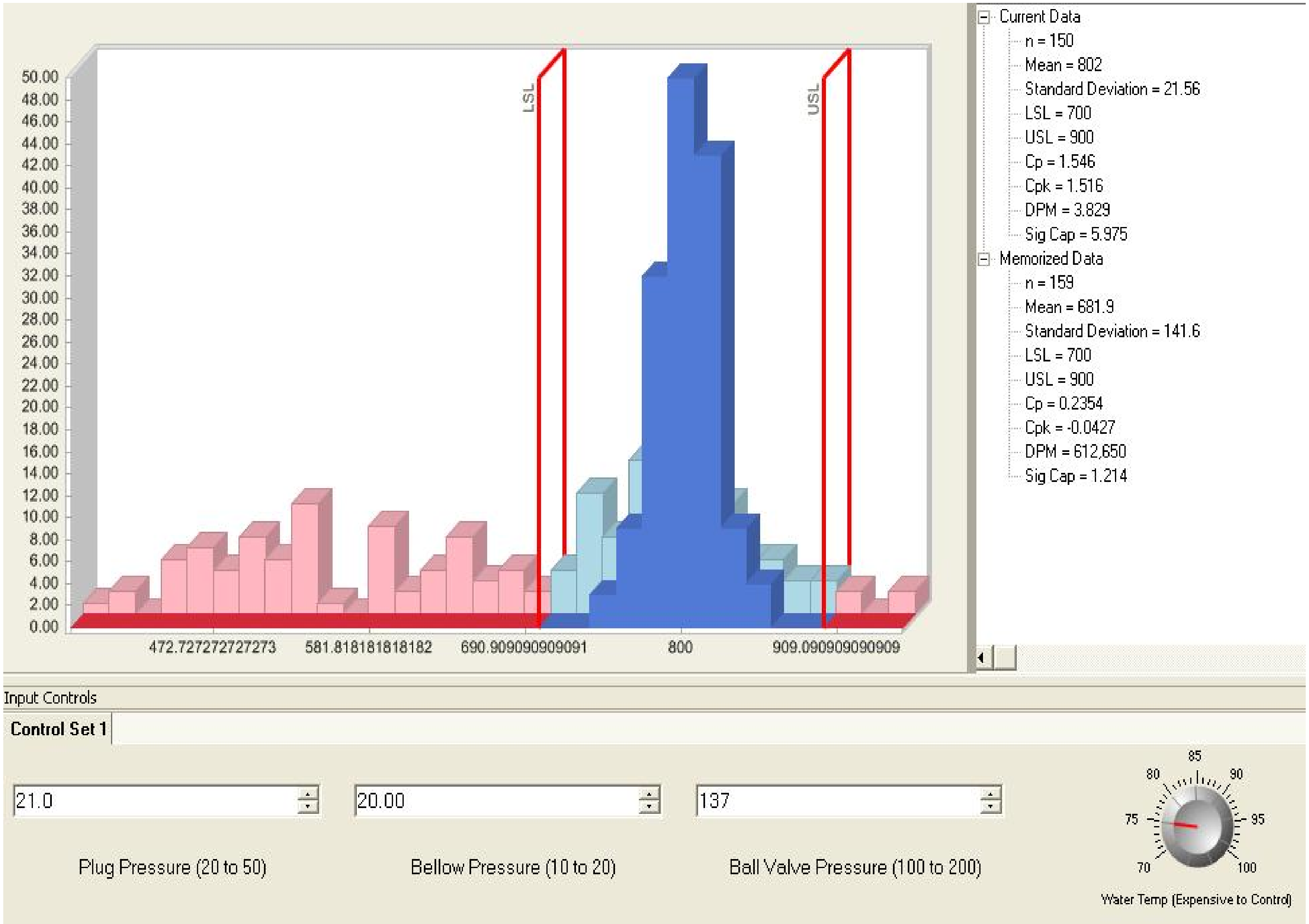
After Robust Design



Current Data
- n = 150
- Mean = 802
- Standard Deviation = 21.56
- LSL = 700
- USL = 900
- Cp = 1.546
- Cpk = 1.516
- DPM = 3.829
- Sig Cap = 5.975

Memorized Data
- n = 159
- Mean = 681.9
- Standard Deviation = 141.6
- LSL = 700
- USL = 900
- Cp = 0.2354
- Cpk = -0.0427
- DPM = 612,650
- Sig Cap = 1.214

Input Controls

Control Set 1

Plug Pressure (20 to 50): 21.0

Bellow Pressure (10 to 20): 20.00

Ball Valve Pressure (100 to 200): 137

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Growth Rate of Factorial Designs

For 2-level designs and k factors: 2^k combinations

- for k = 2 factors: 2^2 = 4 combinations
- for k = 3 factors: 2^3 = 8 combinations
- for k = 10 factors: 2^{10} = 1,024 combinations

For 3-level designs and k factors: 3^k combinations

- for k = 2 factors: 3^2 = 9 combinations
- for k = 3 factors: 3^3 = 27 combinations
- for k = 10 factors: 3^{10} = 59,049 combinations

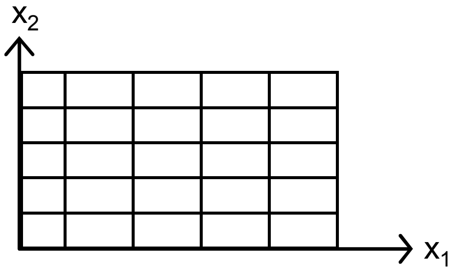
What if the # of factors and/or the number of levels gets large?

Representative Sampling
(Space Filling Designs)

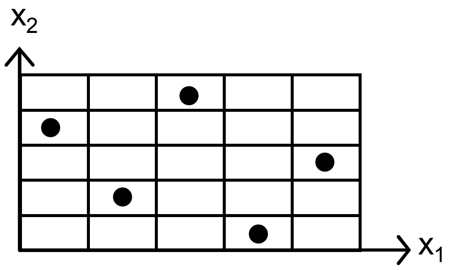
- Method to populate the design space when many variables are involved (e.g., deterministic simulators) or when there are a fixed/limited number of tests specified.
- Design space has k variables (or dimensions).

  Ex: Assume k = 2

- Suppose a sample of size n is to be taken; stratify the design space into n^k cells.
  Ex: Assume n = 5; $n^k = 5^2 = 25$.
- Note: there are n=5 strata for each of the k=2 dimensions.



- Each of the n points is sampled such that each marginal strata is represented only once in the sample.
- Note: each sample point has its own unique row and column.



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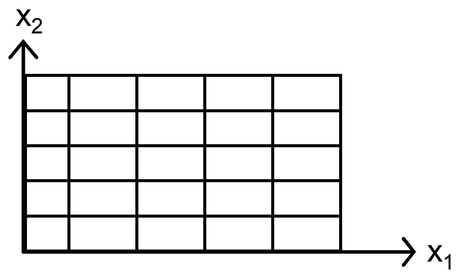
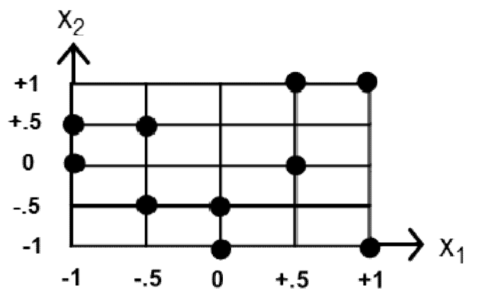
Nearly Orthogonal Latin Hypercube Designs

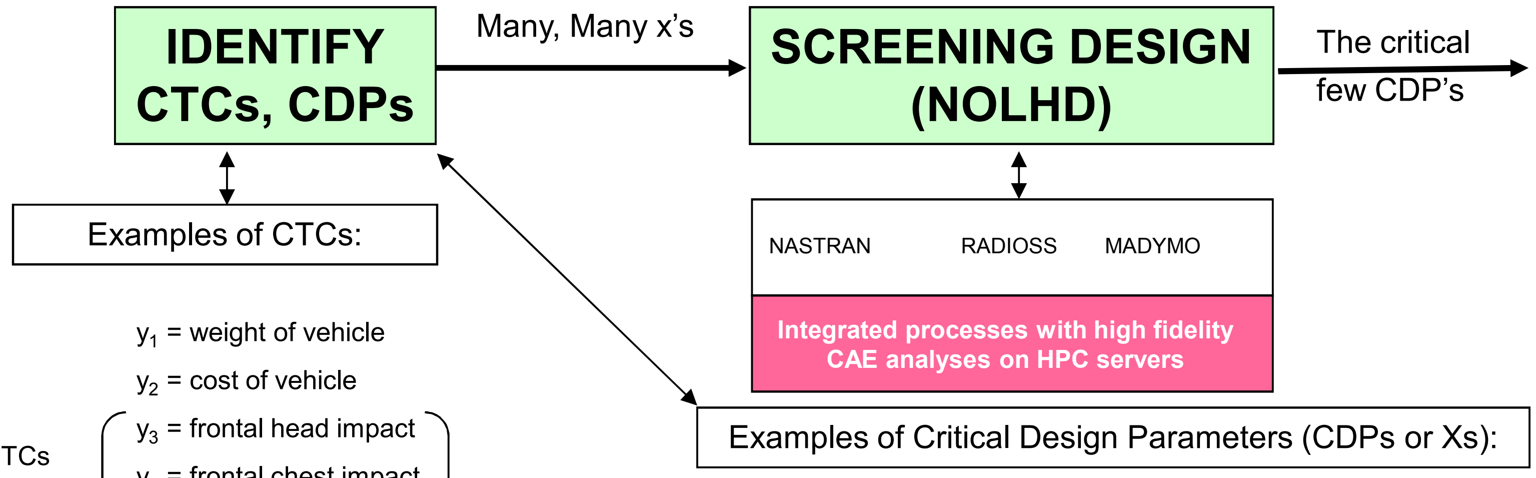
(NOLHD)

- Method to populate a large or high-dimension design space with small samples for the purpose of estimating main effects, quadratic effects, and 2-way interaction effects, as desired.
- Design space has k variables (or dimensions).
  Ex: Assume k = 2.
- User specifies number of levels for each factor.
  Ex: Assume m = 5.
- Total number of sampled data points is n = km or, for this example, n = (2)(5) = 10.
- Each of the n points is selected in such a manner that the resulting design for estimating the desired effects is as orthogonal as possible. This is sometimes called orthogonal space filling, and it will be extremely useful to screen many, many factors.





Applying DOE to Automotive Vehicle Design



IDENTIFY CTCs, CDPs

Many, Many x's

SCREENING DESIGN (NOLHD)

The critical few CDP's

Examples of CTCs:

NASTRAN RADIOSS MADYMO

Integrated processes with high fidelity CAE analyses on HPC servers

Examples of Critical Design Parameters (CDPs or Xs):

y_1 = weight of vehicle

y_2 = cost of vehicle

Safety CTCs with constraints specified by FMVSS (Federal Motor Vehicle Safety Standards)

y_3 = frontal head impact

y_4 = frontal chest impact

y_5 = toe board intrusion

y_6 = hip deflection

y_7 = rollover impact

y_8 = side impact

RADIOSS
DYNA
MADYMO

no federal requirements on these CTCs

y_9 = internal aerodynamics (airflow)

y_{10} = external aerodynamics (airflow)

CFD

y_{11} = noise

y_{12} = vibration (e.g., steering wheel)

y_{13} = harshness (e.g., over bumps, shocks)

y_{14} = durability (at 100K miles)

NASTRAN

x_1 = roof panel material

x_2 = roof panel thickness

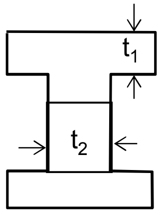
x_3 = door pillar dimensions ⇒ i beam

x_4 = shape/geometry

x_5 = windshield glass

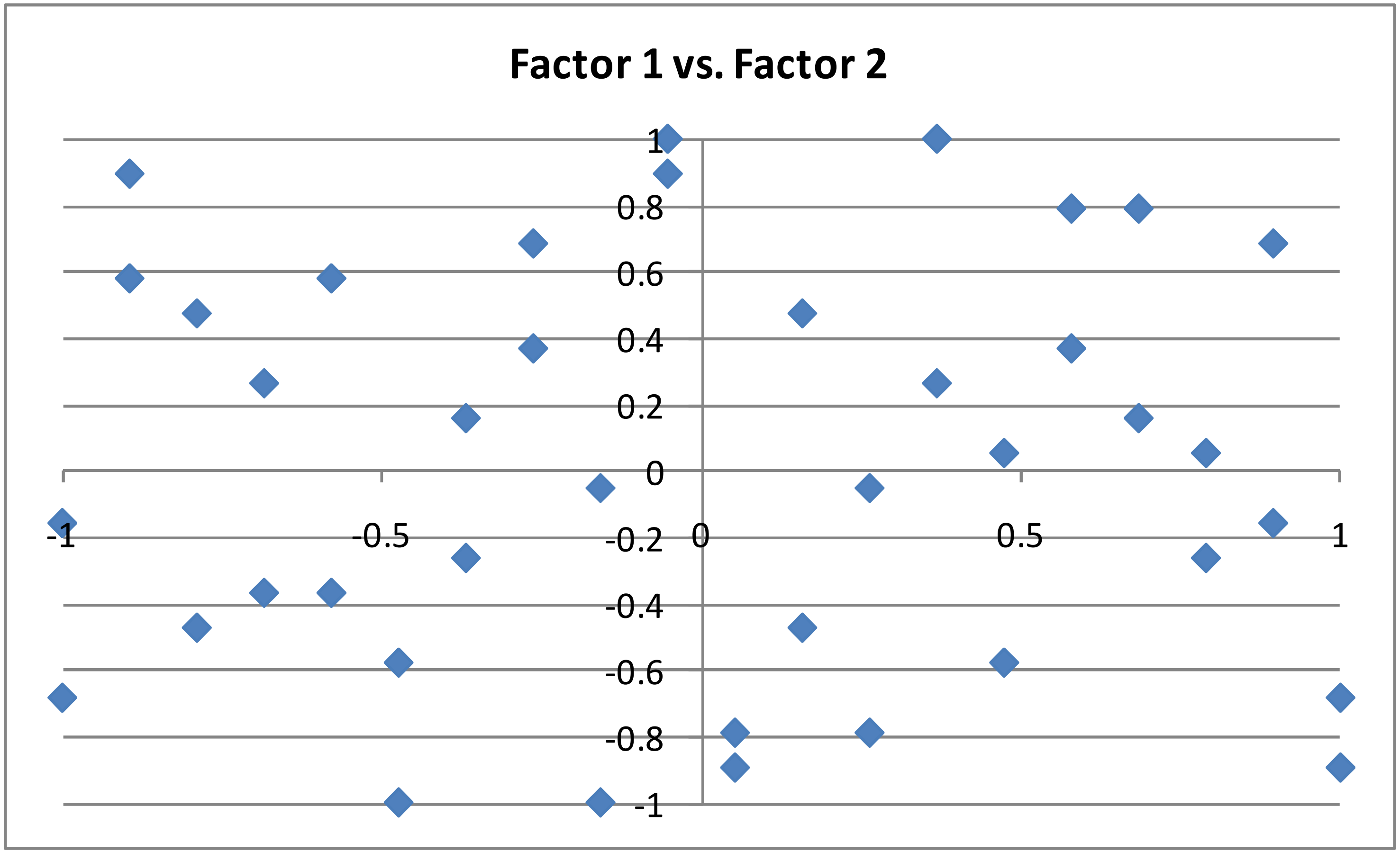
x_6 = hood material, sizing and thickness

x_7 = under hood panel material, sizing and thickness



Nearly Orthogonal Latin Hypercube Design

(20 variables each at 20 levels projected onto x1 vs x2)



Note the balance in the design.

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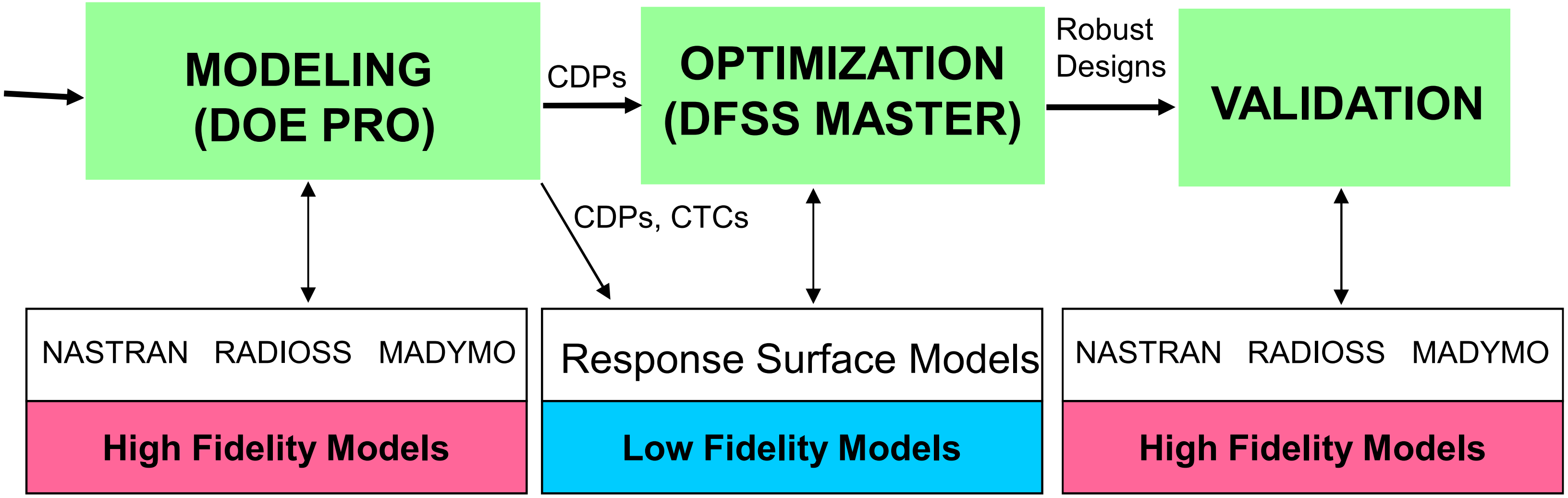
Applying Modeling and Simulation to Automotive Vehicle Design (cont.)



MODELING (DOE PRO)

CDPs

OPTIMIZATION (DFSS MASTER)

Robust Designs

VALIDATION

CDPs, CTCs

NASTRAN RADIOSS MADYMO

High Fidelity Models

Response Surface Models

Low Fidelity Models

NASTRAN RADIOSS MADYMO

High Fidelity Models

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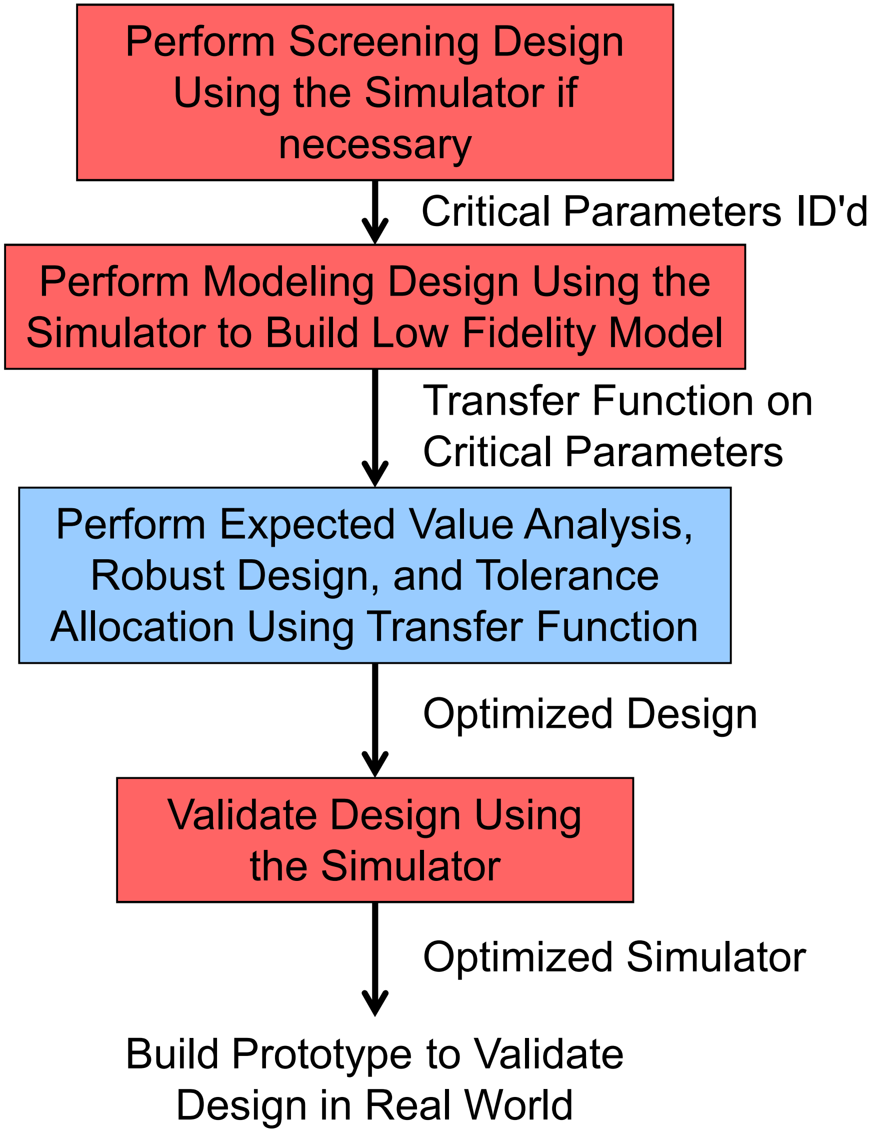
Using DOE to “Optimize the Simulator"



Perform Screening Design Using the Simulator if necessary

Critical Parameters ID'd

Perform Modeling Design Using the Simulator to Build Low Fidelity Model

Transfer Function on Critical Parameters

Perform Expected Value Analysis, Robust Design, and Tolerance Allocation Using Transfer Function

Optimized Design

Validate Design Using the Simulator

Optimized Simulator

Build Prototype to Validate Design in Real World

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Environments Where DOE is Beneficial in Simulation and Modeling

- **A high number of design variables**
- **A substantial number of design subsystems and engineering disciplines**
- **Interdependency and interaction between the subsystems and variables**
- **Multiple response variables**
- **Need to characterize the system at a higher level of abstraction**
- **Time and/or space must be compressed**

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Test Designs for Mixed Factors and Mixed Levels

a.k.a. High Throughput Testing (HTT) or Combinatorial Testing

- A recently developed technique based on combinatorics
- Used to test myriad combinations of many factors (quantitative or qualitative) where the factors could have many levels
- Uses a minimum number of runs or combinations to do this
- Software is needed to select the minimal subset of all possible combinations to be tested so that all 2-way combinations are tested, i.e., all pairs testing
- HTT is not a DOE technique, although the terminology is similar
- A run or row in an HTT matrix is, like DOE, a combination of different factor levels which, after being tested, will result in a successful or failed run
- HTT has its origins in the pharmaceutical business where in drug discovery many chemical compounds are combined together (combinatorial chemistry) at many different strengths to try to produce a reaction.
- Other industries are now using HTT, e.g., software testing, materials discovery, integration, and verification testing (see example on next page).

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All Pairs Testing Example

(Performance Verification and Validation)

- **We would like to perform verification testing with 4 input factors described below.**

- **All possible combinations would involve how many test combinations?**

- **If we were interested in testing all pairs only, how many runs would be in the test matrix and what would those combinations be? To answer this question, we used ProTest software. See next page.**

Sensor Type	Weapon Type	External Data Link	Target Type
S1	**W1**	**Yes**	**T1**
S2	**W2**	**No**	**T2**
S3	**W3**		**T3**
S4			**T4**
			T5

All Pairs Testing Example (cont.)
20 Test Cases

	Sensor	**Weapon**	**Data Link**	**Target**
Case 1	S1	W2	Yes	T1
Case 2	S4	W1	Yes	T2
Case 3	S2	W1	No	T3
Case 4	S3	W3	Yes	T4
Case 5	S2	W3	Yes	T5
Case 6	S4	W3	No	T1
Case 7	S3	W2	No	T2
Case 8	S1	W3	Yes	T3
Case 9	S1	W1	No	T4
Case 10	S3	W1	No	T5
Case 11	S2	W1	No	T1
Case 12	S1	W3	No	T2
Case 13	S4	W2	No	T3
Case 14	S2	W2	Yes	T4
Case 15	S4	W2	No	T5
Case 16	S3	W2	Yes	T3
Case 17	S1	W1	Yes	T5
Case 18	S2	W2	Yes	T2
Case 19	S3	W3	Yes	T1
Case 20	S4	W2	No	T4

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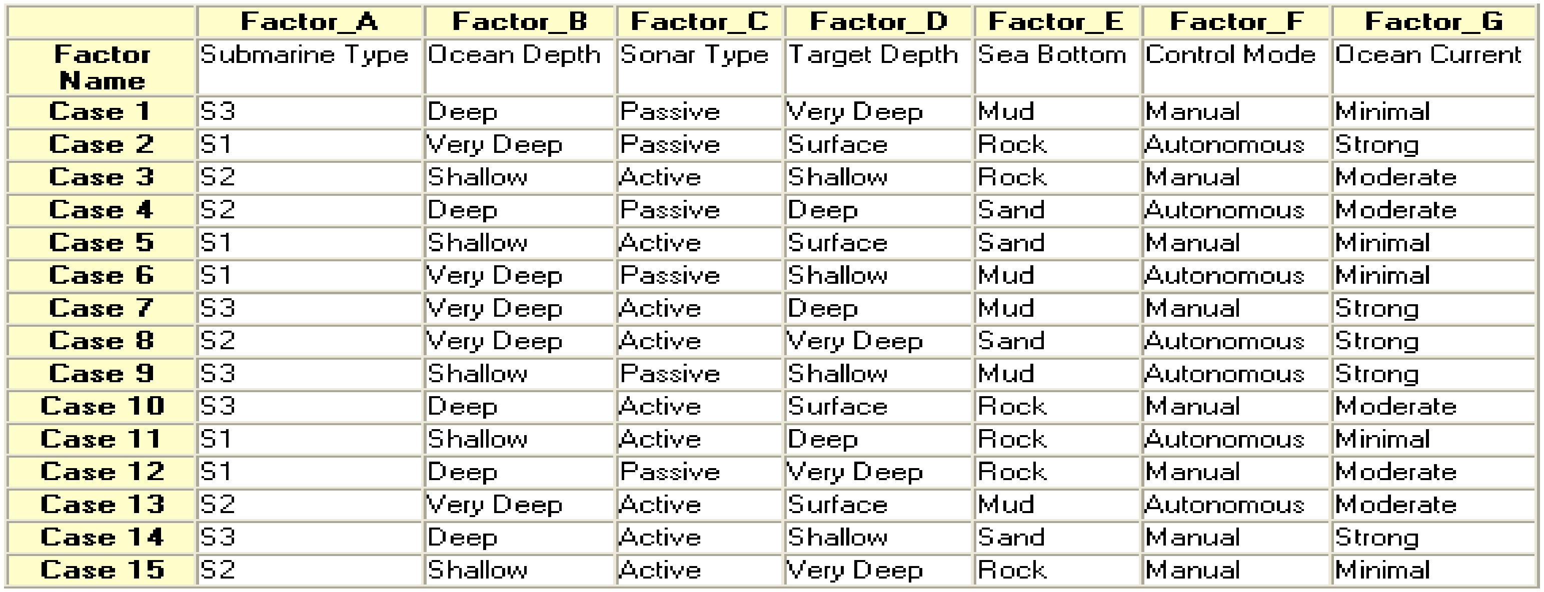
Submarine Threat Detection Test Example

- Suppose we want to perform a verification test with the following 7 input factors (with their respective settings):
  - Submarine Type (S1, S2, S3)
  - Ocean Depth (Shallow, Deep, Very Deep)
  - Sonar Type (Active, Passive)
  - Target Depth (Surface, Shallow, Deep, Very Deep)
  - Sea Bottom (Rock, Sand, Mud)
  - Control Mode (Autonomous, Manual)
  - Ocean Current (Strong, Moderate, Minimal)
- All possible combinations would involve how many runs in the test?
- If we were interested in testing all pairs only, how many runs would be in the test? Pro Test generated the following test matrix.



	Factor_A	**Factor_B**	**Factor_C**	**Factor_D**	**Factor_E**	**Factor_F**	**Factor_G**
Factor Name	Submarine Type	Ocean Depth	Sonar Type	Target Depth	Sea Bottom	Control Mode	Ocean Current
Case 1	S3	Deep	Passive	Very Deep	Mud	Manual	Minimal
Case 2	S1	Very Deep	Passive	Surface	Rock	Autonomous	Strong
Case 3	S2	Shallow	Active	Shallow	Rock	Manual	Moderate
Case 4	S2	Deep	Passive	Deep	Sand	Autonomous	Moderate
Case 5	S1	Shallow	Active	Surface	Sand	Manual	Minimal
Case 6	S1	Very Deep	Passive	Shallow	Mud	Autonomous	Minimal
Case 7	S3	Very Deep	Active	Deep	Mud	Manual	Strong
Case 8	S2	Very Deep	Active	Very Deep	Sand	Autonomous	Strong
Case 9	S3	Shallow	Passive	Shallow	Mud	Autonomous	Strong
Case 10	S3	Deep	Active	Surface	Rock	Manual	Moderate
Case 11	S1	Shallow	Active	Deep	Rock	Autonomous	Minimal
Case 12	S1	Deep	Passive	Very Deep	Rock	Manual	Moderate
Case 13	S2	Very Deep	Active	Surface	Mud	Autonomous	Moderate
Case 14	S3	Deep	Active	Shallow	Sand	Manual	Strong
Case 15	S2	Shallow	Active	Very Deep	Rock	Manual	Minimal

Command & Control Test Example

(15 factors each at various levels)
Total Combinations: 20,155,392

Variable or Factor	Levels	(# of levels)
Mission Snapshots	Entry, Operations, Consolidation	(3)
Network Size	10 Nodes, 50 Nodes, 100 Nodes	(3)
Network Loading	Nominal, 2X, 4X	(3)
Movement Posture	ATH, OTM1, OTM2	(3)
SATCOM Band	Ku, Ka, Combo	(3)
SATCOM Look Angle	0, 45, 75	(3)
Link Degradation	0%, 5%, 10%, 20%	(4)
Node Degradation	0%, 5%, 10%, 20%	(4)
EW	None, Terrestrial, GPS	(3)
Interoperability	Joint Services, NATO	(2)
IA	None, Spoofing, Hacking, Flooding	(4)
Security	NIPR, SIPIR	(2)
Message Type	Data, Voice, Video	(3)
Message Size	Small, Medium, Large, Mega	(4)
Distance Between Nodes	Short, Average, Long	(3)

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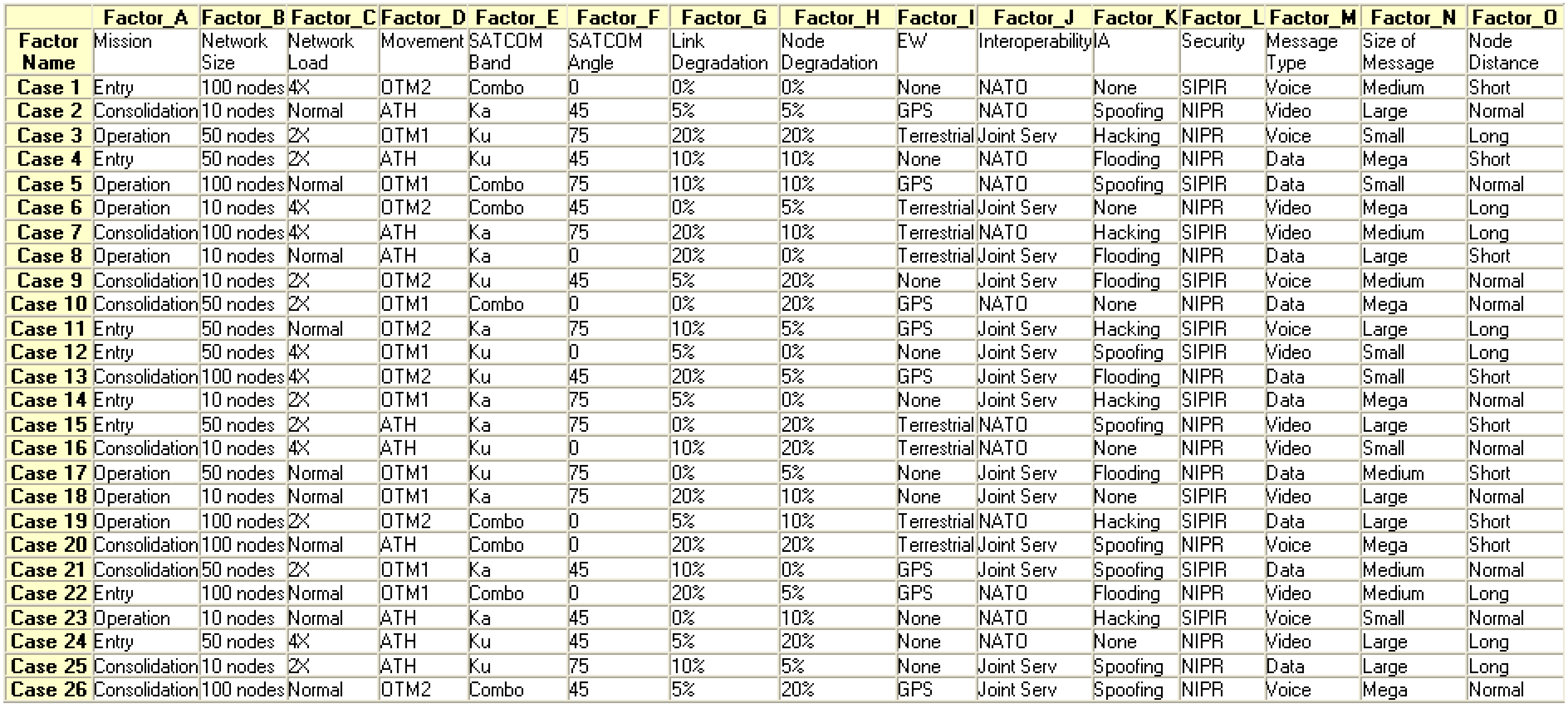
Command & Control Test Example

(All Pairs Testing from ProTest generates 26 test cases)



	Factor_A	Factor_B	Factor_C	Factor_D	Factor_E	Factor_F	Factor_G	Factor_H	Factor_I	Factor_J	Factor_K	Factor_L	Factor_M	Factor_N	Factor_O
Factor Name	Mission	Network Size	Network Load	Movement	SATCOM Band	SATCOM Angle	Link Degradation	Node Degradation	EW	Interoperability	IA	Security	Message Type	Size of Message	Node Distance
Case 1	Entry	100 nodes	4X	OTM2	Combo	0	0%	0%	None	NATO	None	SIPIR	Voice	Medium	Short
Case 2	Consolidation	10 nodes	Normal	ATH	Ka	45	5%	5%	GPS	NATO	Spoofing	NIPR	Video	Large	Normal
Case 3	Operation	50 nodes	2X	OTM1	Ku	75	20%	20%	Terrestrial	Joint Serv	Hacking	NIPR	Voice	Small	Long
Case 4	Entry	50 nodes	2X	ATH	Ku	45	10%	10%	None	NATO	Flooding	NIPR	Data	Mega	Short
Case 5	Operation	100 nodes	Normal	OTM1	Combo	75	10%	10%	GPS	NATO	Spoofing	SIPIR	Data	Small	Normal
Case 6	Operation	10 nodes	4X	OTM2	Combo	45	0%	5%	Terrestrial	Joint Serv	None	NIPR	Video	Mega	Long
Case 7	Consolidation	100 nodes	4X	ATH	Ka	75	20%	10%	Terrestrial	NATO	Hacking	SIPIR	Video	Medium	Long
Case 8	Operation	10 nodes	Normal	ATH	Ka	0	20%	0%	Terrestrial	Joint Serv	Flooding	NIPR	Data	Large	Short
Case 9	Consolidation	10 nodes	2X	OTM2	Ku	45	5%	20%	None	Joint Serv	Flooding	SIPIR	Voice	Medium	Normal
Case 10	Consolidation	50 nodes	2X	OTM1	Combo	0	0%	20%	GPS	NATO	None	NIPR	Data	Mega	Normal
Case 11	Entry	50 nodes	Normal	OTM2	Ka	75	10%	5%	GPS	Joint Serv	Hacking	SIPIR	Voice	Large	Long
Case 12	Entry	50 nodes	4X	OTM1	Ku	0	5%	0%	None	Joint Serv	Spoofing	SIPIR	Video	Small	Long
Case 13	Consolidation	100 nodes	4X	OTM2	Ku	45	20%	5%	GPS	Joint Serv	Flooding	NIPR	Data	Small	Short
Case 14	Entry	10 nodes	2X	OTM1	Ka	75	5%	0%	None	Joint Serv	Hacking	SIPIR	Data	Mega	Normal
Case 15	Entry	50 nodes	2X	ATH	Ka	75	0%	20%	Terrestrial	NATO	Spoofing	NIPR	Video	Large	Short
Case 16	Consolidation	10 nodes	4X	ATH	Ku	0	10%	20%	Terrestrial	NATO	None	NIPR	Video	Small	Normal
Case 17	Operation	50 nodes	Normal	OTM1	Ku	75	0%	5%	None	Joint Serv	Flooding	NIPR	Data	Medium	Short
Case 18	Operation	10 nodes	Normal	OTM1	Ka	75	20%	10%	None	Joint Serv	None	SIPIR	Video	Large	Normal
Case 19	Operation	100 nodes	2X	OTM2	Combo	0	5%	10%	Terrestrial	NATO	Hacking	SIPIR	Data	Large	Short
Case 20	Consolidation	100 nodes	Normal	ATH	Combo	0	20%	20%	Terrestrial	Joint Serv	Spoofing	NIPR	Voice	Mega	Short
Case 21	Consolidation	50 nodes	2X	OTM1	Ka	45	10%	0%	GPS	Joint Serv	Spoofing	SIPIR	Data	Medium	Normal
Case 22	Entry	100 nodes	Normal	OTM1	Combo	0	20%	5%	GPS	NATO	Flooding	NIPR	Video	Medium	Long
Case 23	Operation	10 nodes	Normal	ATH	Ka	45	0%	10%	None	NATO	Hacking	SIPIR	Voice	Small	Normal
Case 24	Entry	50 nodes	4X	ATH	Ku	45	5%	20%	None	NATO	None	NIPR	Video	Large	Long
Case 25	Consolidation	10 nodes	2X	ATH	Ku	75	10%	5%	None	Joint Serv	Spoofing	NIPR	Data	Large	Long
Case 26	Consolidation	100 nodes	Normal	OTM2	Combo	45	5%	20%	GPS	Joint Serv	Spoofing	NIPR	Voice	Mega	Normal

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The Efficiency of All Pairs Testing

- **Suppose we had 75 Factors to test.**
- **Suppose we wanted to test each of these at 2 levels.**
- **How many total combinations are there?**

2^{75} = 37, 778, 931, 862, 957, 161, 709, 568

i.e., 37 Sextillion, 778 Quintillion, 931 Quadrillion, 862 Trillion, 957 Billion, 161 Million, 709 Thousand, 568

- **What is the minimum number of these combinations that will have to be tested in order to test every 2-way combination?**
- **To answer this question, we used our Pro-Test software. The answer is 14 runs or experimental combinations.**
- **For k factors each having the same number of levels tested, say v, then the minimum number of tests ≈ v^2 (ln k)**

Simplify, Perfect, Innovate

Useful Applications of HTT

- **Reducing the cost and time of testing while maintaining adequate test coverage**
- **Integration, functionality, and verification testing**
- **Creating a test plan to stress a product and discover problems**
- **Identifying the critical factors affecting performance in an operational test environment**
- **Prescreening before a large DOE to ensure all 2-way combinations are feasible before discovering, midway through an experiment, that certain combinations are not feasible**
- **Developing an "outer array" of noise combinations to use in a robust design DOE when the number of noise factors and settings is large**

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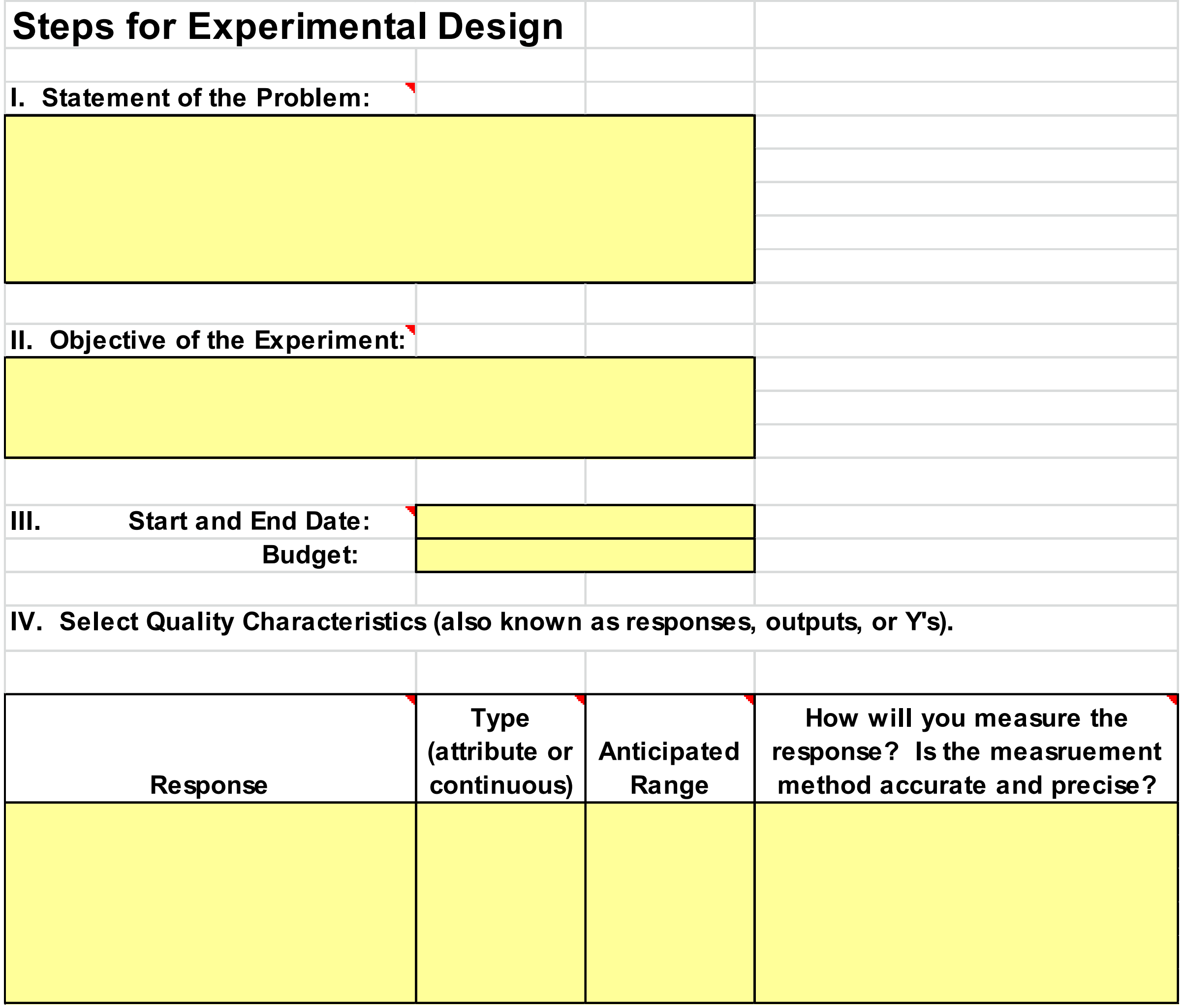
The 12-Step Approach to DOE



Steps for Experimental Design

I. Statement of the Problem:

II. Objective of the Experiment:

III. Start and End Date:

Budget:

IV. Select Quality Characteristics (also known as responses, outputs, or Y's).

Response	Type (attribute or continuous)	Anticipated Range	How will you measure the response? Is the measruement method accurate and precise?

The 12-Step Approach to DOE (cont.)

V. Complete a literature review, process flow diagram, and cause/effect diagram. Select factors which are anticipated to have an effect on the response. Write SOPs for all variables that are to be held constant.

Factor	Type (attribute or continuous)	Controllable or Noise	Range of Interest	Levels	Anticipated Interactions With	How Measured?

VI. Determine the number of resources to be used in the experiment.

VII. Which design types and analysis strategies are appropriate?

VIII. Select the best design type and analysis strategy to suit your needs.

The 12-Step Approach to DOE (cont.)

IX. Can all the runs be randomized? Which factors are most difficult to randomize?

X. Conduct the experiment and record the data.

XI. Analyze the data, draw conclusions, make predictions, and do confirmatory tests.

XII. Assess results, make decisions, and document your results.

Key Take-Aways

- Various approaches to combinatorial test, to include OFAT and Oracle (Best Guess).

- DOE brings orthogonal or nearly orthogonal designs into play.

- Orthogonality (both vertical and horizontal balance in a design) is key to being able to evaluate the effects of factors and their interactions independently from one another.

- Factorial designs are great, but in a world of large test design spaces, we need something else.

- Latin Hypercube Sampling and Descriptive Sampling are useful design strategies when we want good test coverage for many variables with a minimum or specified number of tests. However, these designs are typically not orthogonal.

- Nearly Orthogonal Latin Hypercube Designs provide a sampling strategy to test a large number of factors with a much smaller number of runs than what a factorial design requires, while still retaining adequate orthogonality. These are particularly useful when designing experiments for computer simulations.

- All Pairs Testing, a type of HTT, is a way to get great test coverage (i.e., all pairwise combinations) with a minimal number of runs for a test scenario involving mixed factors (quantitative or qualitative) with a mixed number of levels. This would be a candidate design for OT&E when we are trying to verify and validate performance in an operational envelope. These designs can be orthogonal or nearly orthogonal.

Thank You

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